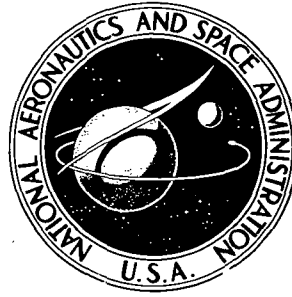


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HIGH-SPEED FLOW PAST WINGS

by Helge Nørstrud

Prepared by
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Marietta, Ga.
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HIGH-SPEED FLOW PAST WINGS

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SUMMARY

The analytical solution to the transonic small-perturbation equation which describes steady compressible flow past finite wings at subsonic speeds can be expressed as a nonlinear integral equation with the perturbation velocity potential as the unknown function. This known formulation is substituted by a system of nonlinear algebraic equations to which various methods are applicable for its solution. Due to the presence of mathematical discontinuities in the flow solutions, however, a main computational difficulty in the present study was to ensure uniqueness of the solutions when local velocities on the wing exceeded the speed of sound. For continuous solutions this was achieved by embedding the algebraic system in an one-parameter operator homotopy in order to apply the method of parametric differentiation. The solution to the initial system of equations appears then as a solution to a Cauchy problem where the initial condition is related to the accompanying incompressible flow solution.

In using this technique, however, a continuous dependence of the solution development on the initial data is lost when the solution reaches the minimum bifurcation point. A steepest descent iteration technique has, therefore, been added to the computational scheme for the calculation of discontinuous flow solutions. Results for purely subsonic flows and supersonic flows with and without compression shocks are given and compared with other available theoretical solutions.

INTRODUCTION

The importance of having aircraft flying in the transonic speed range either in the cruise mode or a maneuvering mode has been expressed by both civilian and military operators. Although the financial benefits from a commercial transonic airplane seems at present to be somewhat questionable, no definite conclusion can be drawn before a thorough understanding has been reached of the various operational problems involved. The technical difficulties, however, associated with the development of such an aircraft are well known to both the experimentalists and the theoretician in the area of predicting the aerodynamic loads and the operating boundaries.

Early attempts to theoretically analyze compressible flow past finite wings at subsonic speeds beyond an application of simple sweep or strip theory were based on the equivalence theorem which concerns itself with geometrically equivalent bodies (ref. 1). A solution was thus obtainable for small aspect ratio wings, but required the solution to the equivalent

nonlifting axisymmetric case. A more general three-dimensional treatment of the transonic flow problem, however, seems to have been first given by Alksne and Spreiter (ref. 2) who extended the concept of local linearization to nonlifting wings, but were limited to oncoming flows which deviated little from the sonic speed (see reference 3 for a more recent and detailed exposition). Similar restriction on the freestream condition also applies to the work of Burg (ref. 4) which is based on still another linearized form of the governing transonic small-perturbation equation. Nørstrud (ref. 5) adopted the integral equation approach to the solution of the same governing nonlinear equation, but results were confined to subcritical flows. The analytical difficulties associated with the three-dimensional transonic flow problem were clearly a matter which was inherited from the more familiar planar case and a solution to the spatial problem dependent to a large extent on the progress made in solving the problem in two dimensions, see e.g. reference 6 for a comprehensive literature review of the subject.

The first successful evaluation of mixed subsonic/supersonic three-dimensional flow with embedded shocks were recently given by Bailey and Steger (ref. 7) who numerically solved the steady state small-perturbation equation using relaxation techniques. Their approach includes a hybrid combination of the perturbation velocity potential and the perturbation velocities as the dependent variables and calculations for lifting, supercritical flows are presented. Similar relaxation schemes have also been applied by Caradonna and Isom (ref. 8) to the transonic flow problem of hovering helicopter blades at zero lift. Still further numerical investigations of the transonic wing alone or wing-body problem are found in references 9, 10 and 11 and this has brought the subject of transonic flow theory to an initial stage of overall completeness.

The present study refines and extends the work of reference 5 to lifting flows with the inclusion of shock discontinuities in three-dimensions. The approach taken follows some fundamental steps proposed for two-dimensional flows by Oswatitsch in 1950 (ref. 12) and it can be described as semi-analytical. The formal integral solution to the transonic small-perturbation equation with associated boundary conditions is regarded as more amenable to numerical analysis than the differential formulation. This is especially true for subcritical flows. For supercritical flows, however, some additional considerations must be given to guarantee the uniqueness of the flow solution. In either case, the governing integral equation is first replaced by a system of nonlinear algebraic equations and then alternative methods of solution to the algebraic system are given.

SYMBOLS

A_i, B_i, C_i	velocity parameters
AR	aspect ratio
Ci	cosine integral
c	speed of sound
$E(x-\xi , Z-\zeta ; r)$	influence function
F_i	functional defined in equation (23)

f_i	functional defined in equation (37)
G_i	operator defined in equation (30)
H_ν	Struve function of integer order ν
J	Jacobian matrix
M	local Mach number
$m(x, Z)$	scaling parameter, see equation (22)
N_ν	Neumann function of integer order ν
$q(\xi, \zeta)$	source distribution, see equation (8)
R	distance function
$r(x, Z)$	velocity decay parameter
S	wing planform area
Si	sine integral
s	specific entropy
U, V, W	transformed perturbation velocity components
u, v, w	velocity components
Y, Z	transformed coordinates = $(1 - M_\infty^2)^{1/2} y, z$
x, y, z	Cartesian coordinates
γ_w	function describing wing surface distribution
Greek Letters:	
α	angle of attack
$\gamma(\xi, \zeta)$	vorticity distribution, see equation (9)
∇^2	Laplace operator
Δ	incremental value
δ_{ij}	Kronecker delta
ϵ_{ij}	influence coefficients
κ	ratio of specific heats
μ	differentiation parameter
ξ, η, ζ	integration variables
ρ, σ, χ	arguments

τ	thickness ratio
Φ	velocity potential
$\phi, \bar{\phi}$	velocity perturbation potentials
ϕ'	downwash correction potential
ψ	functional defined in equation (36)

Subscripts:

∞	freestream value
s	with reference to the source solution
v	with reference to the vortex solution
w	refers to the wing surface
2,3	two-dimensional or three-dimensional respectively

Superscripts:

+	refers to upper surface value
-	refers to lower surface value
x	refers to x-direction
Z	refers to Z-direction

ANALYTICAL ANALYSIS

Governing Differential Equations

Three-dimensional irrotational compressible flow of an ideal gas around a finite wing generates a velocity field $(u, v, w) = \text{grad } \Phi$ which is defined in a body-fixed Cartesian coordinate system x, y, z . With the further assumption of small disturbances in the plane normal to the direction of the oncoming steady flow (i.e., in the yz -plane) the governing partial differential equation for the velocity potential $\Phi = \Phi(x, y, z)$ will be written as

$$(1 - M_\infty^2)\Phi_{xx} + \Phi_{yy} + \Phi_{zz} = (\kappa + 1) \frac{M_\infty^2}{u_\infty} [\Phi_x - u_\infty] \Phi_{xx} \quad (1)$$

where $M_\infty (= u_\infty / c_\infty)$ is the freestream Mach number and κ designates the ratio of specific heats. Equation (1) is a particular form of the set of equations which can be deduced from the transonic small-perturbation theory and it can be transformed to

$$\varphi_{xx} + \varphi_{YY} + \varphi_{ZZ} = \varphi_x \varphi_{xx} \quad (2)$$

after a normalized velocity perturbation potential $\varphi = \varphi(x, Y, Z)$ is defined as

$$\varphi = \frac{(\kappa + 1) M_\infty^2}{(1 - M_\infty^2) u_\infty} (\Phi - u_\infty x - v_\infty y - w_\infty z) \quad (3)$$

The employed coordinate transformation $Y, Z = (1 - M_\infty^2)^{1/2} y, z$ corresponds to a form of the Prandtl-Glauert transformation and we are thus limiting the present analysis to far-fields with subsonic flow ($M_\infty < 1$). This restriction is arbitrary since a different coordinate transformation would make a similar perturbation analysis valid for freestream Mach numbers $M_\infty \geq 1$.

The introduction of a velocity potential Φ in equation (1) is equivalent to imposing the condition of irrotationality

$$\text{rot}(\text{grad } \Phi) = 0 \quad (4)$$

on the flow field and, hence, equations (1) and (4) must be regarded as the system of equations to be solved under given boundary conditions. Since equation (4) implies a characterization of the subject flow as global isentropic ($s = \text{const.}$) and assumes no vorticity in the upstream far field region, the entropy increase across any embedded shock surface is regarded negligible. Thus, the entropy condition which forbids the occurrence of expansion shocks must be incorporated in the analysis by means other than the use of the specific entropy measure. This missing formulation will be dealt with in the numerical analysis section.

Boundary conditions. - Let the planform projection of the wing be situated in the plane

$y = 0$ and let the distribution $y_w^\pm = h^\pm(x, z)$ define the wing upper $(+)$ and lower $(-)$ surface, respectively. The simplified boundary condition expressing tangential flow at the wing surface is then

$$y = \pm 0: \quad \phi_y = u_\infty h_x^\pm(x, z) \quad (5a)$$

or in transformed variables

$$Y = \pm 0: \quad \phi_Y = \frac{(\kappa + 1)M_\infty^2}{(1 - M_\infty^2)^{3/2}} [h_x^\pm(x, z) - \tan \alpha] \quad (5b)$$

where $\alpha (= \tan^{-1}(v_\infty/u_\infty))$ designates the angle of attack which will be limited to small values. Furthermore, equation (5b) assumes $w_\infty = 0$ for simplicity of the analysis. The condition at infinity shall be that of vanishing flow disturbances and will be stated as

$$(x^2 + Y^2 + Z^2)^{1/2} \rightarrow \infty: \quad \phi = 0 \quad (6)$$

In the case of circulatory (or lifting) flows, the Kutta condition at the trailing edge of the wing must be taken into consideration in order to ensure finite velocities of the flow as it leaves the wing surface. The associated potential jump $\Delta\phi$ at the trailing edge varies in a fashion similar to the spanwise load distribution, but remains constant in the streamwise direction in the so-called vortex wake behind the wing.

The harmonic solution. - For the purpose of constructing a solution to the nonlinear partial differential equation (2) we will first consider the problem of obtaining a harmonic solution to the Laplace equation

$$\phi_{xx} + \phi_{YY} + \phi_{ZZ} = 0 \quad (7)$$

This equation is well known from linearized subsonic theory and we can identify a solution $\bar{\phi} = \bar{\phi}(x, Y, Z)$ which satisfies equation (7) and the boundary conditions (5b) and (6) as a transformed Prandtl-Glauert potential with the conversion factor $(\kappa + 1)M_\infty^2(1 - M_\infty^2)^{-1}$, see equation (3).

Analytical solutions to equation (7) for given wing configuration (or boundary conditions) are rare and, in fact, it seems like Holme's solutions, given in reference 13, are the only ones available. These are closed-form incompressible flow solutions for rectangular and triangular wings with parabolic arc cross-sections at zero incidence. Of the various numerical methods available for the evaluation of $\bar{\phi}(x, Y, Z)$, e.g., references 14 and 15, the calculative technique employed in the present study is based on an application of the method of singularity (ref. 16). Since the superposition principle is valid in treating the linear equation (7), this method of solution utilizes a planar distribution of elementary sources and vortices to represent the physical wing configuration and the induced circulatory system.

Thus, for a constant source distribution of strength $q(\xi, \zeta)$ over an area element ΔS in the xZ -plane where $Z = (1 - M_\infty^2)^{1/2} z$ one is faced with an integration of the form (ref. 16)

$$\bar{\phi}_s = -\frac{1}{2\pi} \int_{\Delta S} \int q(\xi, \zeta) \frac{1}{[(x - \xi)^2 + Y^2 + (Z - \zeta)^2]^{1/2}} d\xi d\zeta \quad (8)$$

Similarly, a constant strength vortex distribution $\gamma(\xi, \zeta)$ leads to the representation

$$\begin{aligned} \bar{\phi}_v = & \frac{1}{2\pi} \int_{\Delta S} \int \gamma(\xi, \zeta) \frac{y(x - \xi)}{y^2 + (z - \zeta)^2} \frac{1}{[(x - \xi)^2 + Y + (Z - \zeta)^2]^{1/2}} d\xi d\zeta + \\ & + \frac{1}{2\pi} \int_{\Delta S} \int \gamma(\xi, \zeta) \frac{y}{y^2 + (z - \zeta)^2} d\xi d\zeta \end{aligned} \quad (9)$$

Since the wing planform geometry is necessarily discretized by a number of small planar elements, the appropriate limits of integration are defined by the corner points of the individual panels. Within each of these panel elements the strength of the singularities $q(\xi, \zeta)$ and $\gamma(\xi, \zeta)$ are assumed constant and a control point is selected at which the local boundary conditions are to be satisfied.

The associated computer program which implements the solution of the spherical harmonic $\bar{\phi} = \bar{\phi}_s + \bar{\phi}_v$ of equation (7) for given boundary condition is listed in Appendix B as subroutine CARM. It should be noted that the original listing of this program as obtained from the NASA Langley Research Center has been modified to consider perturbation velocities in the x -direction only.

The Integral Equation Formulation

Equation (2) resembles a Poisson differential equation for which potential theory admits the solution

$$\phi(x, Y, Z) = \bar{\phi}(x, Y, Z) + \phi'(x, Y, Z) - \frac{1}{4\pi} \iiint_{-\infty}^{\infty} [\phi_{\xi\xi} \phi_{\xi\xi}]_{\xi, \eta, \zeta} \frac{1}{R_3} d\xi d\eta d\zeta \quad (10)$$

and where $R_3 = [(x - \xi)^2 + (Y - \eta)^2 + (Z - \zeta)^2]^{1/2}$ designates the inverse of the singular fundamental solution for the domain of integration. To the harmonic solution $\bar{\phi}$ is added another harmonic solution ϕ' which shall serve as a correction potential to the integral term of equation (10). Since the boundary condition (5b) shall also be satisfied by the potential $\bar{\phi}$, the need for the correction term will become apparent when one differentiates equation (10) with respect to Y and rearrange to yield

$$\varphi_Y(x, Y, Z) - \bar{\varphi}_Y(x, Y, Z) = \varphi'_Y(x, Y, Z) - \frac{1}{4\pi} \iiint_{-\infty}^{\infty} [\varphi_{\xi} \varphi_{\xi\xi}]_{\xi, \eta, \zeta} \frac{\partial}{\partial Y} \left(\frac{1}{R_3} \right) d\xi d\eta d\zeta$$

where the kernel

$$\frac{\partial}{\partial Y} \left(\frac{1}{R_3} \right) = - \frac{(Y - \eta)}{[(x - \xi)^2 + (Y - \eta)^2 + (Z - \zeta)^2]^{3/2}}$$

is an antisymmetric function with respect to the xZ-plane for $Y \equiv 0$.

The derivation of equation (10) for symmetrical flows, i.e. with $\varphi'(x, Y, Z) \equiv 0$, has been given by various authors including Gullstrand (ref. 17) who gave a detailed derivation in connection with the Green's theorems. Similar derivation is found in reference 18 and Klunker (ref. 19) generalizes the use of the integral formulation of the transonic small-disturbance velocity potential for the far field boundary condition in connection with finite-difference calculation methods, see e.g. reference 20.

Next, we seek to modify equation (10) for a necessary numerical analysis. If a perturbation velocity vector in the transformed space x, Y, Z is defined as $(U, V, W) = \text{grad } \varphi$ and if we restrict ourselves to seek a solution for $U(x, Y, Z)$ in the planform plane ($Y = 0$) then a differentiation of equation (10) with respect to x will yield after some manipulation

$$U(x, 0, Z) - \frac{1}{2} U^2(x, 0, Z) = \bar{U}(x, 0, Z) + \varphi'_x(x, 0, Z) + \frac{1}{4\pi} \iiint_{-\infty}^{\infty} \frac{U^2}{2}(\xi, \eta, \zeta) \frac{\partial^2}{\partial \xi^2} \left(\frac{1}{R_3} \right) d\xi d\eta d\zeta \quad (11)$$

To facilitate the solution of this nonlinear integral equation, an exponential decay in the Y-direction will be assumed for the perturbation velocity component $U = U(x, Y, Z)$, i.e.

$$U(x, \pm Y, Z) = U(x, \pm 0, Z) \exp\{\mp Y/r(x, \pm 0, Z)\} \quad (12)$$

where the parameter $r^\pm(x, Z) = r(x, \pm 0, Z)$ is approximately derived from equations (4) and (5) as

$$r^\pm(x, Z) = \text{abs} \left\{ (1 - M_\infty^2)^{1/2} \left[\frac{\bar{u}(x, \pm 0, z) - u_\infty}{u_\infty} \right] \middle/ \frac{d^2}{dx^2} (y_w^\pm) \right\}$$

Equation (12) substituted in the integral term of equation (11) will then yield, after an integration in the η -direction, the reduced integral equation

$$\begin{aligned}
U(x, 0, Z) - \frac{1}{2} \dot{U}^2(x, 0, Z) = \bar{U}(x, 0, Z) + \varphi'_x(x, 0, Z) + \frac{1}{4\pi} \iint_{-\infty}^{\infty} U^2(\xi, +0, \zeta) E^+ d\xi d\zeta + \\
+ \frac{1}{4\pi} \iint_{-\infty}^{\infty} U^2(\xi, -0, \zeta) E^- d\xi d\zeta
\end{aligned} \quad (13)$$

where the kernel $E^\pm = E(|x - \xi|, |Z - \zeta|; r^\pm)$ in the double integrals reads

$$\begin{aligned}
E(|x - \xi|, |Z - \zeta|; r^\pm) &= \frac{1}{2} \int_{\eta=0}^{\infty} \exp(-2\eta/r^\pm) \frac{\partial^2}{\partial \xi^2} \left(\frac{1}{R_3} \right) d\eta \\
&= \frac{1}{2} \frac{\partial}{\partial \xi} \int_{\eta=0}^{\infty} \exp(-2\eta/r^\pm) \frac{\partial}{\partial \xi} \left(\frac{1}{R_3} \right) d\eta
\end{aligned} \quad (14)$$

Inserting the relation (valid for $Y \equiv 0$)

$$\frac{\partial}{\partial \xi} \left(\frac{1}{R_3} \right) = \frac{(x - \xi)}{[(x - \xi)^2 + \eta^2 + (Z - \zeta)^2]^{3/2}}$$

in equation (14) and making the substitution

$$t = \eta/r \quad \text{i.e.} \quad d\eta = r dt$$

where the upper superscripts to the parameter r have been dropped one obtains the following expressions

$$\begin{aligned}
E(|x - \xi|, |Z - \zeta|; r) &= \frac{\partial}{\partial \xi} \left\{ \frac{x - \xi}{r^2} \int_{t=0}^{\infty} \exp(-2t) \frac{1}{\left[\left(\frac{x - \xi}{r} \right)^2 + \left(\frac{Z - \zeta}{r} \right)^2 + t^2 \right]^{3/2}} dt \right\} \\
&= \frac{\partial}{\partial \xi} \left\{ \frac{x - \xi}{r^2} \left(-\frac{2\pi}{\sigma} \right) \left[\frac{2}{\pi} - H_1(\sigma) + N_1(\sigma) \right] \right\}
\end{aligned} \quad (15)$$

Here $\sigma = 2 \left[\left(\frac{x - \xi}{r} \right)^2 + \left(\frac{Z - \zeta}{r} \right)^2 \right]^{1/2}$ and the integral is obtained from standard tables, see e.g. reference 21.

from reference 44 one can deduce the following properties

$$\frac{d}{d\sigma} H_1(\sigma) = H_0(\sigma) - \frac{1}{\sigma} H_1(\sigma)$$

$$\frac{d}{d\sigma} N_1(\sigma) = -N_2(\sigma) + \frac{1}{\sigma} N_1(\sigma)$$

for the Neumann function N_ν and the Struve function H_ν of both integer order ν and with the aid of these relations the final evaluation of equation (15) will yield (see figures 1 and 2)

$$\begin{aligned} E(|x - \xi|, |Z - \zeta|; r) = & -\frac{2\pi}{r^2 \sigma} \left\{ \left[\frac{4}{\sigma^2} \left(\frac{x - \xi}{r} \right)^2 - 1 \right] \left[\frac{2}{\pi} - H_1(\sigma) + N_1(\sigma) \right] + \right. \\ & \left. + \frac{4}{\sigma} \left(\frac{x - \xi}{r} \right)^2 \left[H_0(\sigma) - \frac{1}{\sigma} (H_1(\sigma) + N_1(\sigma)) + N_2(\sigma) \right] \right\} \quad (16) \end{aligned}$$

which reduces to

$$E^Z = -\frac{2\pi}{r\sigma} \left[\frac{2}{\pi} - H_1(\sigma) + N_1(\sigma) \right] \quad (17)$$

$$E^x = \frac{2\pi}{r} \left[H_0(\sigma) - \frac{1}{\sigma} (H_1(\sigma) + N_1(\sigma)) + N_2(\sigma) \right] \quad (18)$$

for $x - \xi \equiv 0$ and $Z - \zeta \equiv 0$ respectively. Equation (17) has previously been obtained in reference 5, but omits the function r in the denominator as a result of a typographical error.

Correction for lifting, continuous flows. - In the case of lifting flows, the integral terms of equation (13) will give rise to a downwash in the plane of the wing planform. Since the potential $\bar{\varphi}$ is already required to satisfy the specified boundary conditions in this plane, i.e., $\bar{\varphi}_Y = \varphi_Y$ at $Y = \pm 0$, the correction potential φ' is added to the harmonic solution $\bar{\varphi}$ to counterbalance this induced downwash. Furthermore, φ' must also be a harmonic function, but we can confine this requirement (or property) to the xY -plane since equation (13) does not depend on any spanwise (i.e., approximately the Z -direction) derivatives of the function φ' . We will first consider continuous flow and define a correction potential $\varphi' = \varphi'(x, Y, Z)$ which shall be antisymmetric with respect to the planform plane $Y = 0$ as

$$\varphi'(x, Y, Z) = -\frac{1}{\pi} \int_{\xi=-\infty}^{\infty} \gamma'(\xi, Z) \tan^{-1} \frac{Y}{x - \xi} d\xi \quad (19)$$

The vortex strength $\gamma'(x, Z)$ is necessarily defined as

$$\gamma'(x, Z) = \frac{1}{2} [U(x, +0, Z) - \bar{U}(x, +0, Z) - U(x, -0, Z) + \bar{U}(x, -0, Z)] \quad (20)$$

and a differentiation of equation (19) with respect to x together with the limit value

$$\gamma'(x, Z) = \lim_{Y \rightarrow 0} \frac{1}{\pi} \int_{\xi} \gamma'(\xi, Z) \frac{Y}{(x - \xi)^2 + Y^2} d\xi$$

yields the identity $\varphi'_x(x, 0, Z) = \gamma'(x, Z)$. Since a unique circulation is already implemented in writing equation (20) no further considerations have to be made regarding the Kutta condition at the trailing edge.

Correction for lifting, discontinuous flows. - With shock surface discontinuities in the flow field a modification of equation (20) is necessary in order to assure the harmonic nature of $\varphi'(x, 0, Z)$ and, hence, enforce a smooth continuous derivative φ'_x . However, instead of defining the function φ' we define its gradient in the x -direction as

$$\varphi'_x(x, 0, Z) = \frac{m(x, Z)}{4} [\bar{U}^2(x, +0, Z) - \bar{U}^2(x, -0, Z)] \quad (21)$$

and let $m(x, Z)$ represent a spanwise-varying scaling parameter which shall depend on the local circulation around the wing in order to satisfy the Kutta condition. Hence, one can write

$$m(x, Z) = 2 \int_{\xi=-\infty}^{\infty} [U(\xi, +0, Z) - \bar{U}(\xi, +0, Z) - U(\xi, -0, Z) + \bar{U}(\xi, -0, Z)] [\bar{U}^2(\xi, +0, Z) - \bar{U}^2(\xi, -0, Z)]^{-1} d\xi \quad (22)$$

It should be emphasized that equation (21) is not unique and that a better downwash representation at the leading edge could be achieved with, say, an adoption of the Riegel's factor $[1 + (\partial y_w / \partial x)^2]^{-1/2}$.

From the elementary relation

$$\bar{U}^2(x, +0, Z) - \bar{U}^2(x, -0, Z) = [\bar{U}(x, +0, Z) - \bar{U}(x, -0, Z)] [\bar{U}(x, +0, Z) + \bar{U}(x, -0, Z)]$$

where the first and second factor on the right hand side represents the linearized perturbation velocity due to lift and thickness respectively. One sees from equation (21) that the distribution in chordwise direction of the x -component of the correction potential is defined to be of a similar form as the disturbance due to thickness. This distribution, however, is modified by an amount proportional to the local chordwise lift. It should be noted that the use of a different combination of the linearized velocity components in equation (21) could be better defined from known lifting flow calculations.

NUMERICAL ANALYSIS

The Algebraic System

The solution to the nonlinear integral equation (13) cannot be obtained analytically for wing configuration of arbitrary planform and thickness distribution. Hence, a numerical treatment of the general problem is mandatory and the first step in this direction is to discretize the area of integration in accord with the division of the wing planform as used for the numerical evaluation of the harmonic solution. The range of integration shall be confined to the wing planform and the nonlinear compressibility sources outside the cylinder which encloses the planform and which have the generatrix parallel to the Y-direction are therefore not being considered.

Let the upper and lower surface of the left (or right) half of the wing planform each be represented by N number of trapezoidal panels and let the unknown function $U = U(x, \pm 0, Z)$ assume a constant value within these specified panel boundaries, see figure 3. Then equation (13) can be replaced by a system of $2N$ algebraic equations which are nonlinear (i.e., quadratic) with respect to $U = U(x, 0, Z)$, i.e.

$$F_i(U_1, U_2, \dots, U_{2N}) = U_i - \frac{1}{2} U_i^2 - \bar{U}_i + C_i + \sum_{j=1}^{2N} \epsilon_{ij} U_j^2 = 0 \quad i = 1, 2, \dots, 2N \quad (23)$$

where the index $i \leq N$ shall identify a value on the upper surface of the wing (or panel). Consequently, the lower surface values are associated with the range $N < i \leq 2N$.

Due to the lateral symmetry assumed for the wing planform and its position relative to the oncoming flow (i.e. $w_\infty = 0$) only one half of the wing need to be considered in the algebraic system (23) since the influence from two panels which are positioned symmetrical with respect to the x-axis can be lumped together as $\epsilon_{ij} = \epsilon_{ij_1} + \epsilon_{ij_2}$. (Note that $U_{i_1} = U_{i_2}$ where the indices 1 and 2 refers to each half of the wing.)

The influence coefficients ϵ_{ij} are integral representations of the influence function $E = E(|x - \xi|, |Z - \zeta|; r)$ as evaluated over a surface element ΔS , i.e. of the form

$$\frac{1}{4\pi} \iint_{\Delta S} E(|x - \xi|, |Z - \zeta|; r) d\xi d\zeta \rightarrow \epsilon_{ij}$$

for $|x - \xi| \neq 0$ and $|Z - \zeta| \neq 0$. Due to the particular and perhaps unfortunate influence pattern of positive and negative contributions from the integral terms of equation (13) care must be exercised in the numerical evaluation of the coefficients ϵ_{ij} , see figure 4.

This is further illustrated by the series representations

$$E^X(\rho) = -4 \left[\frac{1}{2} - \frac{2}{3} \rho + \frac{4}{45} \rho^2 - \dots \right] \quad |\rho| \rightarrow 0 \quad (24)$$

$$E^Z(\chi) = 4 \left[\frac{1}{2} - \frac{1}{\chi} + \frac{1}{3} \chi - \frac{1}{45} \chi^3 + \dots \right] \quad |\chi| \rightarrow 0 \quad (25)$$

of the influence function E in chordwise and approximate spanwise directions respectively where $\rho = 2|Z - \xi|/r$ and $\chi = 2|x - \xi|/r$. Since both equations (24) and (25) show similar behavior at the singular points $\rho = \chi = 0$, but of opposite sign, we let $\epsilon_{ii} = 0$. Furthermore, we define the influence coefficients ϵ_{ij} as

$$\epsilon_{ij} = \frac{\Delta S}{4} [E^{(1)} + E^{(2)} + E^{(3)} + E^{(4)}] \quad (26)$$

Equation (26) is derived from a simple application of the mean value theorem of integral calculus by relating the influence strength ϵ_{ij} at the centroid of each panel to the influence measure at four points $l = 1, 2, 3, 4$ located on the four panel boundary lines (see figure 5). A better approximation of ϵ_{ij} would be achieved, however, if a surface spline function is utilized in connection with the four values $E^{(l)}$ and the local value of E at the centroid, see e.g. reference 23. Note that for symmetrical flow problems it is sufficient to consider only a system of N equations with the minor modification of doubling the value of the influence coefficients ϵ_{ij} ($i, j = 1, 2, \dots, N$). However, the following analysis will be presented for a system of $2N$ equations.

The function C_i ($i = 1, 2, \dots, 2N$) appearing in equations (23) is a velocity correction function associated with circulatory flows and is easily obtained from equation (20) as

$$C_i = \frac{1}{2} A_i \quad i = 1, 2, \dots, 2N \quad (27)$$

for continuous flow. For circulatory flows with a shock discontinuity, equation (21) approximately defines the required correction as

$$C_i = \frac{1}{2} B_i \sum_{j=1}^N (1 - \delta_{ij}) \frac{A_j}{B_j} \quad i = 1, 2, \dots, 2N \quad (28)$$

where δ_{ij} represents the Kronecker delta and where

$$A_i = \begin{cases} U_i - \bar{U}_i - U_{i+N} + \bar{U}_{i+N} & i \leq N \\ U_i - \bar{U}_i - U_{i-N} + \bar{U}_{i-N} & i > N \end{cases}$$

and

$$B_i = \begin{cases} \bar{U}_i^2 - \bar{U}_{i+N}^2 & i \leq N \\ \bar{U}_i^2 - \bar{U}_{i-N}^2 & i > N \end{cases}$$

In the case of non-circulatory flows, equations (27) and (28) will yield the expected corrections of $C_i \equiv 0$. System (23) is now completely determined and the task of finding a vector $U_i (i = 1, 2, \dots, 2N)$ which satisfies equations (23) for given ϵ_{ij} and \bar{U}_i will be the subject of the following section.

Methods of Solution

Solutions to the nonlinear system (23) could be obtained by various iteration or relaxation schemes and each method has its own merits and limitation. The decisive factor in selecting a solution technique, however, is the type of flow under consideration, see figure 6. Since we are also interested in obtaining discontinuous solutions which shall represent flows with compression shocks, we will distinguish between three different types of flow problems, namely

- o Subcritical flow
- o Supercritical flow without discontinuities
- o Supercritical flow with discontinuities

and describe some numerical approaches for their unique solution. In essence, each one of the numerical methods to be introduced should yield the same solution to the quadratic equations (23) for identical given flow configurations. This is especially correct in the subcritical case where only the smallest roots of system (23) are sought (i.e., all U_i -values corresponds to subsonic velocities) and this single solution is necessarily unique.

However, a supercritical discontinuous solution vector $U_i (i = 1, 2, \dots, 2N)$ of system (23) is signified by being a combination of roots which does not belong to the same family, i.e. subcritical or supercritical where the critical value $U_i = 1$ correspond to the case of two equal roots. Hence, multivalued solutions are possible and some additional condition must be introduced in the calculation scheme in order to render the obtained solution unique. Such a condition is in our case a substitute for the entropy inequality

which forbids the occurrence of expansion shocks in real flows, and the present study has adopted a version of the method of steepest descent to be a suitable calculation scheme in which the "entropy" condition can be introduced. This is achieved by an approximate predetermination of the location of the critical points ($U_i = 1$) as indicated by the linearized solution \bar{U}_i ($i = 1, 2, \dots, 2N$) and this again forces the calculation scheme to search for one particular solution.

An intermediate class of transonic flow solutions are characterized by being both continuous and supercritical in the sense that subsonic and supersonic velocities can exist together within a certain control volume without being separated by a surface of discontinuity. Both theoretical and experimental investigations made in the more recent years have confirmed this type of flow phenomena and the introduction of the method of parametric differentiation in the present study can be viewed upon as an attempt to calculate and to plausibly explain such flows. This has been achieved by embedding the solution to the system (23) in a Cauchy type problem in which a value of $U_i = 3/2$ appears as the upper critical value for a unique dependence of the initial data upon the sought solution.

In summary, it should be emphasized that the economics of finding a numerical solution of the stated system of equations (23) has been considered in the selection of a suitable algorithm. Thus, the fast converging method of Newton is found to be far the best choice for a subcritical flow calculation despite the fact that the other two solution techniques could be applied as well. These two methods, on the other hand, represent choices for the calculation of the more intricate supercritical flow cases and the ambitious user will find ample opportunities to make improvements of the listed computer program.

The Newton's method. - An application of Newton's method of successive approximation to the solution of system (23) will generate a sequence of vectors $U_i^{\{k+1\}}$ from the relation

$$U_i^{\{k+1\}} = U_i^{\{k\}} - [F_{U_i}^{\{k\}}(U_i)]^{-1} F^{\{k\}}(U_i) \quad i = 1, 2, \dots, 2N \quad (29)$$

where the integer in the bracket $\{ \}$ identifies the sequence number and where the starting vector $U_i^{\{k=0\}}$ can be chosen to be identical to the known vector \bar{U}_i . This method is particularly valuable for subcritical flow calculations, i.e. $U_i^{\{k\}} < 1$ ($i=1, 2, \dots, 2N$), for which the guarantee of uniqueness of a solution belongs to a classical proof in fluid mechanics. (See e.g. reference 24 and its list of references). As long as the functional determinant (or Jacobian) is nonvanishing and positive for all values $U_i < 1$, the necessary inversion of the Jacobian matrix $J(U_i) = F_{U_i}(U_i)$ or

$$J(U_1, U_2, \dots, U_{2N}) = \partial(F_1, F_2, \dots, F_{2N}) / \partial(U_1, U_2, \dots, U_{2N})$$

can be handled by numerous available computer subroutines.

Method of parametric differentiation. - A continuous application of Newton's method to the solution of system (23) for values of U_i exceeding unity (i.e. for super-critical flow), would imply some arbitrariness with regard to the uniqueness or the usefulness (expansion shocks can occur) of the obtained solution. For our purpose in controlling the development of an iterative unique and continuous solution to the system for the case of $U_i > 1$ we will adopt the method of parametric differentiation (see e.g. reference 25) which has found wide applications in mathematical analysis, including a treatment of the transonic flow problem in two dimensions (ref. 26).

Our method of solution will be based on the particular one-parameter operator homotopy (embedding)

$$G_i(\mu; U_1, U_2, \dots, U_{2N}) = U_i - \bar{U}_i + \mu[F_i(U) - U_i + \bar{U}_i] \quad i = 1, 2, \dots, 2N \quad (30)$$

where μ is a real-valued parameter in the range $0 \leq \mu \leq 1$. From the implicit function theorem as applied to equations (30) one obtains the differential properties

$$\frac{dU_i}{d\mu} = -J^{-1}(\mu; U_1, U_2, \dots, U_{2N}) \frac{\partial G_i}{\partial \mu} \quad i = 1, 2, \dots, 2N \quad (31)$$

where the Jacobian matrix of the system is

$$J(\mu; U_1, U_2, \dots, U_{2N}) = \partial(G_1, G_2, \dots, G_{2N}) / \partial(U_1, U_2, \dots, U_{2N})$$

The first-order nonlinear ordinary differential equations (31) together with the initial conditions

$$\mu = 0: U_i = \bar{U}_i \quad i = 1, 2, \dots, 2N \quad (32)$$

defines a Cauchy problem which at $\mu = 1$ yields our sought solution of the algebraic system (23). There exists various powerful methods for the numerical solution of system (31), see for example reference 27, and we will confine ourselves to the use of Hamming's modified predictor-corrector method together with a fourth order Runge-Kutta method for the computation of the starting values. Since the right-hand side of equations (31) includes the parameter μ , the system (31) was solved at every appropriate stage of the integration ($0 \leq \mu \leq 1$) by including a modified Gaussian elimination procedure.

A condition for a unique solution of system (31) in the range $0 \leq \mu \leq 1$ is the non-vanishing of the functional determinant (or Jacobian) of $G_i(\mu; U_1, \dots, U_{2N})$ with respect to U_i for fixed μ . This determinant is positive under the condition that the elements on the principal diagonal are positive, i.e. from equations (23), (27), and (30) we obtain

$$1 - \mu\left(U_i - \frac{1}{2}\right) > 0 \quad i = 1, 2, \dots, 2N \quad (33)$$

Since the initial solution U_i is necessarily continuous and will yield a positive Jacobian, one can conclude by virtue of inequality (33) that a unique and continuous solution of system (31) can at least exist for $U_i < 3/2$ ($i = 1, 2, \dots, 2N$).

However, it is more consistent with the present analysis to express this limit for continuous flow directly in terms of a maximal local perturbation velocity

$$\max \left\{ \left(\frac{u - u_\infty}{u_\infty} \right)_i \right\} = \frac{3(1 - M_\infty^2)}{2(\kappa + 1)M_\infty^2}$$

and then apply, for example, the isentropic relation $M = M(\kappa, u/u_\infty, M_\infty)$ for the evaluation of the corresponding Mach number. The dependence between this maximal local Mach number and the Mach number at infinity is given in figure 7. It is seen that the predicted limitation of shock-free transonic flow is in general accordance with the two-dimensional results from Nieuwland's theory (ref. 28) and of Korn (ref. 29) despite our underlying assumption of small disturbances. Also depicted in the figure is the stability criterion of Spee (ref. 30) which limits the value of the local Mach number to $(5/2)^{1/2}$ for $\kappa = 7/5$.

The method of steepest descent. - For a certain value of the parameter μ the Jacobian matrix in system (31) might become singular and the solution curve to the Cauchy problem, equations (31) and (32), can split into two or more solutions. Such a point $\mu = \mu^*$ in the interval $0 < \mu \leq 1$ for which $\det J(\mu; U_1, U_2, \dots, U_{2N}) = 0$ can be regarded as a bifurcation point and the associated solution $U_i^* = U_i(\mu^*)$ will be designated the bifurcation solution. Since the next step in the integration of system (31) would involve the crossing of a line of a singular Jacobian matrix, a continuous dependence of the solution on the initial data is no longer possible. The condition that the bifurcation solution U_i^* satisfies equation (30) could be utilized in connection with the introduction of a family of differentiation parameters in order to continue the solution curve past the singular lines. This difficult problem, however, will not be further pursued in the present report. Instead, we will apply the method of steepest descent to the solution of system (23) which represents a set of quadratic equations with respect to U_i ($i = 1, 2, \dots, 2N$) and the roots of the system for the case of discontinuous flow are easily obtained as

$$U_{i,2} = 1 + \operatorname{sgn} \left\{ 1 - 2 \left[\bar{U}_i - C_i - 2 \sum_{j=1}^{2N} \epsilon_{ij} U_j^2 \right] \right\}^{1/2} \quad i = 1, 2, \dots, 2N \quad (34)$$

where $\operatorname{sgn} = \pm 1$ depends upon $U_i \geq 1$. Hence, a bifurcation of the solution to equations (34) subject to the initial data $U_i = \bar{U}_i$ occurs when the discriminants vanish and this is equivalent

to considering the identity of the relations

$$\sum_{j=1}^{2N} \epsilon_{ij} U_j^2 \geq \frac{1}{2} \left(\bar{U}_i - C_i - \frac{1}{2} \right) \quad i = 1, 2, \dots, 2N \quad (35)$$

Furthermore, the inequality of (35) gives the condition for positive discriminants and hence the condition for real-valued roots of the algebraic system.

Let a function $\psi = \psi(f_1, f_2, \dots, f_{2N})$ be defined as

$$\psi = \sum_{j=1}^{2N} f_j(U_1, U_2, \dots, U_{2N}) \quad (36)$$

where

$$f_i(U_1, U_2, \dots, U_{2N}) = 1 - U_i + \operatorname{sgn} \left\{ 1 - 2 \left[\bar{U}_i - C_i - 2 \sum_{j=1}^{2N} \epsilon_{ij} U_j^2 \right]^{1/2} \right\} \quad i = 1, 2, \dots, 2N \quad (37)$$

which is obtained from a rewriting of equations (34). In order to find the roots of system (37) the method of steepest descent considers the variational problem of searching for the zero value of ψ from equation (36). The calculation scheme in the method produces a sequence of vectors $\{U_i\}$ from the relations

$$U_i^{\{k+1\}} = U_i^{\{k\}} - \lambda^{\{k\}} \frac{\partial \psi(f_1^{\{k\}}, f_2^{\{k\}}, \dots, f_{2N}^{\{k\}})}{\partial U_i} \quad i = 1, 2, \dots, 2N \quad (38)$$

where the step length λ along the gradient direction of the function ψ is determined at each sequence number k as

$$\lambda^{\{k\}} = \frac{\psi^{\{k\}}}{\sum_{j=1}^{2N} \left[\frac{\partial \psi^{\{k\}}}{\partial U_j} \right]^2}$$

The sign convention in system (37) is either determined from the condition $U_i^{\{k\}} \geq 1$; $\operatorname{sgn} = \pm 1$ or from some information which yields the approximate geometric location of the sonic line and the foot of the shock discontinuity.

The present method utilizes the linearized solution \bar{U}_i ($i = 1, 2, \dots, 2N$) to approximately define any supersonic region on the wing surface or in other words to enforce the sign convention in equations (37). This is accomplished by mapping the region for

$$\bar{U}_i > - \frac{(\kappa + 1)M_\infty^2}{1 - M_\infty^2} c_p^*$$

where c_p^* is the critical pressure coefficient as obtained from the exact isentropic relation and then shift the upstream boundary line one chordwise panel length downstream. The resulting region is then our definition for the area of supersonic velocities and a positive sgn-value in system (37) shall only be connected to points U_i which lies within this region. An improvement of this rather arbitrary technique of defining a sonic line and a line of discontinuity has been attempted with a partial use of the associated bifurcation solution, but required computation times have made such an improvement impractical.

RESULTS AND DISCUSSION

Two-Dimensional Flows

As first examples of continuous flow, subcritical pressure distributions past a nonlifting and lifting NACA 0012 airfoil have been calculated using the Newton's method and are shown in figures 8 and 9, respectively. Comparisons are made with "exact" numerical solutions, see reference 31, which are obtained from an application of Sells method to the full two-dimensional gasdynamic equation. Furthermore, it is found that the present transonic small-perturbation solution as seen in figure 9 is very similar to the solution given by Krüpp (ref. 32) which also assumes small disturbances to the freestream condition.

Figures 10 and 11 show results for supercritical flow past the same airfoil configurations as given in the two preceding figures, but the present solutions are continuous in the sense that supersonic velocities exist on a portion of the profiles without being terminated by a shock. These results are obtained by the method of parametric differentiation and are compared with solutions which have been calculated with the Jameson program, see e.g. reference 33. It is seen that these latter solutions exhibits small compression shocks, at least in the solution given in figure 11. For freestream Mach numbers slightly higher than those given in the figures, no continuous solution could be obtained. This occurrence is interpreted as the Mach number limit for which shock discontinuities start forming on the airfoils.

The result for discontinuous flow past an 8.4 percent ($\tau = 0.084$) thick symmetrical parabolic arc airfoil at zero angle of attack is given in figure 12 and compared with the numerical solutions of references 20 and 34. The solution associated with the latter

reference is, for the given thickness parameter, related to a freestream Mach number of $M_\infty = 0.8496$. The lifting case of a discontinuous solution is depicted in figure 13 for a NACA 0012 airfoil and comparison is made with the numerical results of reference 35.

Three-Dimensional Flows

To test the Woodward/Carmichael computer program which calculates the linearized solution to the posed wing problem, a subsonic lifting case for a rectangular wing of parabolic arc cross-section has been evaluated and plotted in figure 14. The chordwise pressure distribution at various spanwise stations (identified by the spanwise coordinate \bar{z} which is normalized to the semispan) are in good agreement with the results of figure 10 from reference 7. The familiar singular behavior of the solution at the leading edge region, however, is apparent. This is also brought out in figure 15 which compares the calculated incompressible flow solution at mid-span for a high aspect ratio wing with a blunt leading edge with the exact two-dimensional data (reference 36). A further test of the accuracy of the linearized solution is depicted in figure 16 which compares the numerically obtained results for a symmetrical wing with the analytical solution of Holme (ref. 13). It is somewhat surprising to see that the numerical solution yields higher perturbation velocities than the analytical. The reason for this might be in the low number of panels (i.e., 100) used to represent the semi-wing planform in the numerical calculation scheme.

Figures 17 and 18 compares the present subcritical results, as obtained from an application of Newton's method, for a high aspect ratio non-lifting and lifting wing respectively with the "exact" two-dimensional values given in reference 31. For this aspect ratio of 7 the flow at the midspan of the wing should be nearly planar and figure 18 also partly validates the downwash correction potential (see equation 19) employed for this case. The sensitivity, however, of the number of chordwise and spanwise panels used in the calculation scheme is brought out in both figures. Similar results for the same non-lifting wing, but of smaller aspect ratio, is given in figure 19.

For an arbitrary wing configuration the planform of a trapezoidal half-wing is defined by its four corner points and the profile geometry is input in a table-form. This is illustrated by the swept wing configuration of figures 20 and 21. The cross-section of this wing is a RAE 101 airfoil and the profile surface coordinate, slope, and curvature at various chordwise stations are tabulated in subroutine SECTIN as ZTAB1, ZPTAB1, CUTAB1, and XTAB respectively. Calculated supercritical results were obtained by the method of parametric differentiation and compared with the experimental data of reference 35. Both types of data indicates that the flow solutions are continuous.

Discontinuous flows are obtained with an adoption of the method of steepest descent and sample flow calculations are given in figures 22 through 25. The first two figures show results for a non-lifting parabolic arc wing at two different spanwise stations. Comparisons with the numerical solutions of reference 7 are not very encouraging and this

is even more true for the lifting case as shown in figure 24. However, if the discontinuous supersonic region is confined to only a small part of the wing (e.g. in the root section area) a better agreement of the results are found (figure 25).

CONCLUDING REMARKS

The compressibility effects on the flow past lifting and nonlifting wings at high subsonic Mach numbers have been studied with an application of the transonic integral equation method. This semi-analytical calculation method preassumes a solution for the equation describing linearized flow around the same wing configuration, i.e. assumes the availability of the Prandtl-Glauert solution, and this problem is solved by means of known analytical or numerical solutions. The aim of the study was to incorporate the nonlinear compressibility effects in a pressure calculation scheme which also should include the case of a shock compression surface in the flow field.

The present calculation method involves a summation of some compressibility effects which are distributed over the wing planform area and which are a function of local velocity and profile curvature. Since these nonlinear effects can attain different sign convention an approximate numerical integration over the domain of influence depends largely on the grid size employed. An alternative integration technique (approximate two-directional) which involves one sign convention for the influence measure only, has therefore been developed and applied to all calculation cases shown in this report (see figure 3).

The practical desirable case of shock-free supercritical flow was studied by an incorporation of the method of parametric differentiation in the numerical evaluation of the governing algebraic system. Obtained results show that local supersonic velocities can indeed exist in the flow field without a discontinuous recompression in the form of a shock surface. It is felt that these findings constitute an important contribution to the subject involved.

For the calculation of supercritical flows with shock discontinuities, however, the present methods seems less effective. The reason for this lies in the fact that points on the sonic line represent extreme values for the stream-density quantity and an accurate influence definition in the neighborhood of these points is required. Also, a different iteration scheme for the evaluation of the nonlinear equations could greatly improve the usefulness of the presented analysis technique.

APPENDIX A - BASIC EQUATIONS FROM TWO-DIMENSIONAL ANALYSIS

For wings of infinite aspect ratio the analysis becomes two-dimensional and available results for this case can be utilized in the calculation of flows about finite wings (see reference 5). The appropriate integral equation (as compared to equation (10) on page 7) will then read

$$\varphi(x, Y) = \bar{\varphi}(x, Y) + \varphi'(x, Y) - \frac{1}{2\pi} \iint_{-\infty}^{\infty} [\varphi_{\xi} \varphi_{\xi\xi}]_{\xi, \eta} \ln \frac{1}{R_2} d\xi d\eta \quad (A1)$$

where $R_2 = [(x - \xi)^2 + (Y - \eta)^2]^{1/2}$. A similar assumption for the decay of the perturbation velocities in the Y-direction as made in the foregoing three-dimensional analysis will reduce equation (A1) to an equation involving the unknown quantity $U = U(x, \pm Y)$ of the form

$$U(x, 0) - \frac{1}{2} U^2(x, 0) = \bar{U}(x, 0) + \varphi'_x(x, 0) + \frac{1}{2\pi} \int_{-\infty}^{\infty} U^2(\xi, \pm 0) E_2(|x - \xi|; r) d\xi \quad (A2)$$

where the two-dimensional influence function $E_2 = E_2(|x - \xi|; r)$ is given as

$$\begin{aligned} E_2(\chi) &= E_2\left(\frac{2|\xi - x|}{r}\right) \\ &= \frac{4}{\pi} \left\{ \sin(\chi) \left[\frac{\pi}{2} - \text{Si}(\chi) \right] - \cos(\chi) \text{Ci}(\chi) \right\} \end{aligned} \quad (A3)$$

Here Si and Ci are the sine and cosine integrals respectively.

An analogous numerical integration of equation (A2) would involve the determination of influence coefficients ϵ_{ij} and this leads to a definition of

$$\epsilon_{ii} = \frac{1}{2r} \int_0^{\Delta x/2} E_2 d\xi$$

which yields after a substitution of equation (A3) in the integrand of equation (A2) the following value

$$\begin{aligned}\epsilon_{ij}(\chi_o) &= \epsilon_{ij}\left(\frac{\Delta x}{r}\right) \\ &= \frac{1}{\pi} \left\{ \cos(\chi_o) \left[\text{Si}(\chi_o) - \frac{\pi}{2} \right] - \sin(\chi_o) \text{Ci}(\chi_o) + \frac{\pi}{2} \right\}\end{aligned}$$

The values of the coefficients ϵ_{ij} (where $j \neq i$), however, is determined direct from the influence function E_2 as $\epsilon_{ij} = \Delta x E_2(|x - \xi|; r) / (4r)$. These influence coefficients are used in the approximate integration of the nonlinear influence measure over the wing, see e.g. figure 3.

APPENDIX B - LISTING OF COMPUTER PROGRAM

A computer program has been written in the FORTRAN IV language to implement the numerical calculation schemes described in the foregoing sections. The program consists of a main calling program designated MAIN and 17 external subroutine or function subprograms. Subprograms which are part of the particular Carmichael computer program received from the NASA Langley Research Center are designated by an asterisk (*). A flow chart showing the calling sequence of the program subroutines are given in figure 26.

<u>Name</u>	<u>Description</u>
MAIN	Main program
CYL	A subroutine subprogram which evaluates the cylindrical functions N_ν (Neumann function or Bessel function of the second kind) and H_ν (Struve function) both of integer order ν .
SICI	A subroutine which evaluates the sine and cosine integrals (Si and Ci)
ACSH	A function subprogram to evaluate the transcendental function $y = \operatorname{arcsinh} x$ (arcus sine hyperbolicus).
GAUSS	A subroutine which solves a system of linear algebraic equations by a modified Gaussian elimination method.
HPCG	A subroutine to solve a system of first-order ordinary differential equations
FCT	A subroutine which defines the differential system (31)
OUTP	A subroutine used for control of output from HPCG
CARM*	A subroutine used as the calling program for the evaluation of the arbitrary harmonic or linearized solution (i.e. a modified Carmichael program).
WNGEOM*	A subroutine to evaluate the geometric characteristic of the wing.
FILL*	A subroutine for function interpolation in connection with WNGEOM.

SECTIN*	A subroutine used to input the profile geometry. Note: The four data tables XTAB, ZTAB1, ZPTAB1, and CUTAB1 given in this subroutine are for a RAE 101 airfoil and they represent 19 chordwise values for the profile chordwise station together with the associated measure for the thickness, slope and curvature with reference to a profile thickness ratio of one. For arbitrary profile geometry appropriate input data tables must be supplied by the user. If the number of chordwise input stations are different from 19, a dimension statement and three sequential calling statement for subroutine TAIN* must also be changed accordingly.
TAINT*	A subroutine used in connection with SECTIN to interpolate geometric data.
EVAL*	A subroutine to evaluate the aerodynamic influence coefficients in connection with the harmonic solution.
COMP* (TCOMP)	A subroutine used for the calculation of the linearized perturbation velocities due to thickness. This subroutine has an alternative entry point identified by the name TCOMP.
INVERT*	A subroutine for the matrix inversion of the influence matrix evaluated in EVAL.
FORCE*	A subroutine used for the calculation of the complete linearized solution in terms of perturbation velocities.
VECON	A subroutine which converts local perturbation velocities to local Mach numbers, pressure coefficient and pressure ratio.

Listing of main calling program and associated external subroutines.

```

C      MAIN PROGRAM
C
C      THIS PROGRAM CALCULATES THE TRANSONIC PRESSURE DISTRIBUTION
C      ON ARBITRARY LIFTING WINGS AT SUBSONIC SPEEDS
C      BASED ON A (NONLINEAR) INTEGRAL EQUATION METHOD.
C
C      THE PROGRAM WAS LARGELY DEVELOPED UNDER CONTRACT NAS1-10665
C      SPONSORED BY THE LOADS DIVISION OF THE NASA LANGLEY
C      RESEARCH CENTER, BUT INCORPORATES THE WOODWARD/CARMICHAEL
C      COMPUTER PROGRAM FOR THE CALCULATION OF THE LINEARIZED SOLUTION.
C
C      THE PROGRAM IS DIMENSIONED FOR A TOTAL OF 100 PANELS
C      TO REPRESENT THE WING PLANFORM AND THIS LIMITS THE NUMBER
C      OF PANELS WHICH CAN BE USED FOR THE LIFTING CASE TO 50.
C      A SYMMETRICAL NONLIFTING WING, HOWEVER, CAN BE REPRESENTED
C      BY A MAXIMUM OF 100 PANELS. THE STORAGE REQUIREMENT
C      IS APPROXIMATELY 145 000 IN OCTAL.
C
C      DIMENSION
1     IDX(100), IDZ(100), U(100),    UOL(100), FINT(100),
2     DFU(100), PRMT(5), EP(4),     DELTX(50), AUX(16,100),
3     XSU(50), XSL(50), XXU(50),   XXL(50)
C
C      COMMON /DY/ DERY(100),Y(100),UL(100),DP
C      COMMON /HOL/ EPS(10000), A(10100)
C      COMMON /PRNDTL/ UPG(100),ILIFT,NRUN,SYMF
C      COMMON /VEL/ MACH,MACHSQ,BETASQ,BETA,AAM1,AAM2,AAM3
C      COMMON /PARAMS/ NWIN,PANELS,SREF,REFMOM,CBAR,SPAN,OC
C      COMMON /WNGPMS/ ROOT(4),TIP(4),M,N,TYPE,F(101),G(101),P(101),
1     SHEAR(101),ISECT,THICK(5)
C      COMMON/XZW/XW(100),ZW(100)
C      COMMON /CURV/ CUR(100)
C      REAL MACH,MACHSQ,BETASQ,BETA
C      NAMELIST /FLOW/ MACH,BELM,NDELM
C      NAMELIST /WING/ IWIN,INFLU,METHOD,ROOT,TIP,THICK,ISECT,LX,LZ,
1     SPAN,TAU
C      NAMELIST /SHOCK/ XSU,XSL,XXU,XXL
C
C      CONSTANTS AND THE INPUT
C
C      IT=1
C      ACC=0.001
C      PI=3.1415926535
C      PID=2.0*PI
C      PI4=4.0*PI
C      PI83=8.0*PI/3.0

```

```

PIH=1.5707963
PI22=0.3183099
PI2P=0.5/PI
PI2=0.63661977
PI3=3.0*PI
PI1=1.0/PI
PI4I=1.0/PI4
TYPE=4
OC=2

```

C

```

10 READ(5,WING)
   IF (IHING.EQ.9) GO TO 9999
   READ (5,FLOW)
   ILIFT=0
   IF (IHING.GT.0) GO TO 100
   SPAN=2.0*TIP(3)
   SEMI=SPAN/2.0
   ROT=ROOT(2)-ROOT(1)+TIP(2)-TIP(1)
   SREF=TIP(3)*ROT
   KEFMOM=0.25*(ROOT(2)-ROOT(1))
   CBAR=ROT/2.0
   TAU=THICK(1)
   AR=SPAN*SPAN/SREF
   GO TO 110
100 TAUP=TAU/PI
   S=SPAN/2.0
   SEMI=S
   TAU5=TAU*SEMI
110 MACHSQ=MACH*MACH
   BETASQ=1.-MACHSQ
   BETA=SQRT(ABS(BETASQ))
   AAM1=10.0/(7.0*MACHSQ)
   AAM2=0.2*MACHSQ
   AAM3=0.7*MACHSQ
   AM=MACH
   N=LX
   M=LZ
   LXZ=LX*LZ
   LL=LXZ+LXZ
   LL2=LL*LL
   LX22=LXZ*LXZ
   DELTZ=SEMI/FLOAT(LZ)
   DL22=DELTZ/2.0
   NRUN=0

```

C

C

C

THE PROGRAM HEADER

```

200 WRITE (6,210)
210 FORMAT (11H1,27X,36H VELOCITY AND PRESSURE DISTRIBUTION ,
1 40HON AN ARBITRARY WING AT SUBSONIC SPEEDS /42X,
2 46H BASED ON A TRANSONIC INTEGRAL EQUATION METHOD ////)
   WRITE (6,WING)
   WRITE (6,FLOW)

```

C

C

C

THE PRANDTL-GLAUERT SOLUTION

```

215 AM2=AM*AM
   BET2=1.0-AM2
   BETA=SQRT(BET2)

```

```

      IW1=IWING+1
      GO TO (460,217,420), IW1
217  AR=2.0*SEMI
      DLTX=1.0/FLOAT(LX)
      DO 230 NZ=1,LZ
        DELTX(NZ)=DLTX
        DLX2=DLTX/2.0
        DO 220 NX=1,LX
          K=(NZ-1)*LX+NX
          ZW(K)=-DLZ2+FLOAT(NZ)*DELTZ
          XW(K)=-DLX2+FLOAT(NX)*DLTX
220  CONTINUE
230  CONTINUE
      DO 300 NZ=1,LZ
        NN=(NZ-1)*LX+1
        SMZ=(S-ZW(NN))*BETA
        SPZ=(S+ZW(NN))*BETA
        DO 300 NX=1,LX
          K=(NZ-1)*LX+NX
          X=2.0*XW(NX)-1.0
          XP1=1.0+X
          XM1=1.0-X
          B1=ACSH(XP1/SPZ)
          B2=ACSH(XM1/SPZ)
          B3=ACSH(XP1/SMZ)
          B4=ACSH(XM1/SMZ)
          B5=ACSH(SPZ/XM1)
          B6=ACSH(SMZ/XM1)
          B7=ACSH(SPZ/XP1)
          B8=ACSH(SMZ/XP1)
          DUM=SPZ*(B1+B2)+SMZ*(B3+B4)-X*(B5+B6-B7-B8)
          UPG(K)=TAUP*DUM/BETA
          CUR(K)=-4.0*TAU
          IDX(K)=NX
          IDZ(K)=NZ
300  CONTINUE
      GO TO 480
420  AR=4.0*SEMI
      DO 330 NZ=1,LZ
        DLTX=(S-FLOAT(NZ-1)*DELTZ-DLZ2)/S
        DELTX(NZ)=DLTX/FLOAT(LX)
        DLX2=DELTX(NZ)/2.0
        DO 320 NX=1,LX
          K=(NZ-1)*LX+NX
          ZW(K)=-DLZ2+FLOAT(NZ)*DELTZ
          XW(K)=1.0-DLTX+FLOAT(NX)*DELTX(NZ)-DLX2
320  CONTINUE
330  CONTINUE
      S=S*BETA
      SS=S*S
      S0=1.0+SS
      S1=(1.0+2.0*SS)/(S*S0)
      S2=2.0/S
      S3=2.0*S/S0
      S4=1.0/SQRT(S0)
      S5=1.0/S
      X0F=(1.0-DLX2)/S
      DO 450 NZ=1,LZ
        NN=(NZ-1)*LX+1

```

```

Z=ZW(NN)*BETA
ZZ=Z*Z
SPZ=S+Z
SMZ=S-Z
Z1=SMZ*SMZ
Z3=SPZ*SPZ
A8=2.0*Z
DO 450 NX=1,LX
K=(NZ-1)*LX+NX
X=XW(K)
XX=X*X
X1=(1.0-X)**2
XS2=2.0*S*X
XZ=SQRT(XX+ZZ)
A1=(S*X-Z)/S0
A2=SPZ-XS2
A4=(S*X+Z)/S0
A5=SMZ-XS2
SXZM=S*X-Z
SXZP=S*X+Z
V1=(1.0-X+S*SMZ)/SXZM
V2=(X+S*Z)/SXZM
V3=(1.0-X+S*SPZ)/SXZP
V4=(X-S*Z)/SXZP
X10=1.0-X
V5=SMZ/X10
V6=SPZ/X10
V7=Z/X10
B6=ACSH(V1)
B7=ACSH(V2)
B9=ACSH(V3)
B10=ACSH(V4)
SH5=ACSH(V5)
SH6=ACSH(V6)
SH7=ACSH(V7)
B2=SQRT(X1+Z1)+SQRT(X1+Z3)
B3=S2*SQRT(X1+ZZ)
B4=S3*XZ
B5=S4*(A1-A2)
B8=S4*(A4-A5)
B11=A2*SH5
B12=A5*SH6
B13=A8*SH7
DUM=S1*B2-B3-B4+B5*(B6+B7)+B8*(B9+B10)+S5*(B11+B12+B13)
UPG(K)=TAUP*DUM/BETA
CUR(K)=-4.0*TAU
IDX(K)=NX
IDZ(K)=NZ
450 CONTINUE
GO TO 480
460 CALL CARM
DO 470 NZ=1,LZ
NN=(NZ-1)*LX+1
NN1=NN+1
DELTX(NZ)=XW(NN1)-XW(1,N)
DO 470 NX=1,LX
K=(NZ-1)*LX+NX
IDX(K)=NX
IDZ(K)=NZ

```



```

470 CONTINUE
480 CONTINUE
  NS=LL
  IF (ILIFT.EQ.0) NS=LXZ
  NS2=NS*NS
  IF (ILIFT.EQ.0) GO TO 500
  DO 490 I=1,LXZ
    II=I+LXZ
    XW(II)=XW(I)
    ZW(II)=ZW(I)
    CUR(II)=CUR(I)
    IDX(II)=IDX(I)
    IDZ(II)=IDZ(I)
    IF (LWING.EQ.0) GO TO 490
    UPG(II)=UPG(I)
490 CONTINUE

```

```

C
C   THE REDUCED VARIABLES
C

```

```

500 NIT=0
  AM7=0.7*AM2
  AM22=0.2*AM2
  AMF=10.0/(7.0*AM2)
  XM=AM*SQRT(1.0/(1.0+0.16666666*(AM2-1.0)))
  CPST=2.0/(1.4*AM2)*(((2.0+0.4*AM2)/2.4)**3.5-1.0)
  FAK=2.4*AM2
  FAT=BET2
  RY=BETA
  RU=FAK/FAT
  RV=RU/RY
  UCR=-RU*CPST/2.0
  AMT=(2.4*AM2)**0.33333333/TAU**0.66666666
  EMR=(AM2-1.0)/(2.4*TAU*AM2)**0.66666666
  HMT=FAK**0.33333333/TAU**0.66666666
  HMT=HMT/AMT
  DO 800 I=1,NS
    UL(I)=RU*UPG(I)
    U(I)=UL(I)
    UOL(I)=U(I)
800 CONTINUE

```

```

C
C   THE INFLUENCE MATRIX
C

```

```

  IF (INFLU.EQ.0) GO TO 5800
  IF (INFLU.EQ.2) GO TO 2305
  DO 2300 I=1,NS
    DIZI=DELTZ*FLOAT(IDZ(I))-DLZ2
  DO 2290 J=1,NS
    DIZJ=DELTZ*FLOAT(IDZ(J))-DLZ2
    K=(J-1)*NS+I
    IREF=IDZ(J)
    ER=ABS(UPG(J)/CUR(J))*RY
    ER2=ER*ER
    IJ=IABS(I-J)
    IF (IJ.EQ.0.OR.IJ.EQ.LXZ) GO TO 2090
    GO TO 2100
2090 S=ABS(DELTZ(IREF)/ER)
    CALL SICI(S,SINT,CINT)
    EPS1=PI2P*(PIH*(1.0-COS(S))+COS(S)*SINT-SIN(S)*CINT)

```

```

      GO TO 2250
2100 CONTINUE
      IF (IDZ(I).EQ.IDZ(J)) GO TO 2110
      GO TO 2200
2110 DIN=ABS(FLOAT(IDX(I))-IDX(J))
      DISX=DELT(X(IREF))*DIN/ER
      S=ABS(2.0*DISX)
      CALL SICI(S,SINT,CINT)
      EPS1=PI2P*(SIN(S)*(PIH-SINT)-COS(S)*CINT)
      EPS(K)=DELT(X(IREF))*EPS1/ER
      GO TO 2290
2200 CONTINUE
      IF (IDX(I).EQ.IDX(J)) GO TO 2205
      GO TO 2280
2205 NEP=1
2210 DIN=ABS(DIZI-DIZJ)
      IF (NEP.EQ.2) DIN=DIZI+DIZJ
      DISZ=BETA*DIN/ER
      DIX=ABS(XW(I)-XW(J))
      DISX=DIX/ER
      DX2=DISX*DISX
      DISR=SQRT(DX2+DISZ*DISZ)
      DS=2.0*ABS(DISR)
      DS2=DS*DS
      CALL CYL(DS,FH0,FH1,FN1,FN2)
      G1=PI/DS
      G2=PI2-FH1+FN1
      G3=FH0-(FH1+FN1)/DS+FN2
      EFUNC=G1*((4.0*DX2/DS2-1.0)*G2+4.0*DX2*G3/DS)
      EPF=BETA*DELT(X(IREF))*EFUNC/(PI4*ER2)
      IF (NEP.EQ.2) GO TO 2260
      EPS1=EPF
2250 NEP=2
      GO TO 2210
2260 EPS2=EPF
      EPS(K)=EPS1+EPS2
      GO TO 2290
2280 EPS(K)=0.0
2290 CONTINUE
2300 CONTINUE
      GO TO 2600
C
2305 DO 2500 I=1,NS
      DIZI=DELTZ*(FLOAT(IDZ(I))-DLZ2)
      DO 2400 J=1,NS
      DIZJ=DELTZ*(FLOAT(IDZ(J))-DLZ2)
      K=(J-1)*NS+I
      IJ=IABS(I-J)
      IREF=IDZ(J)
      DIX=ABS(XW(I)-XW(J))
      ER=ABS(UPG(J)*RY/CUR)
      ER2=ER*ER
      DO 2390 JJ=1,4
      EPS1=0.0
      IF (IJ.EQ.0.OR.IJ.EQ.LXZ) GO TO 2308
      GO TO 2350
2308 NEP=1
2310 ADD=0.0
      IF (JJ.EQ.1) ADD=-DELT(X(IREF))/2.0

```

```

      IF (JJ.EQ.3) ADD=DELTx(IREF)/2.0
      DISX=(DIX+ADD)/ER
      DX2=DISX*DISX
      DIZ=ABS(DIZI-DIZJ)
      IF (NEP.EQ.2) DIZ=DIZI+DIZJ
      ADD=0.0
      IF (JJ.EQ.2) ADD=DLZ2
      IF (JJ.EQ.4) ADD=-DLZ2
      DISZ=BETA*(DIZ+ADD)/ER
      DISR=SQRT(DX2+DISZ*DISZ)
      DS=2.0*ABS(DISR)
      DS2=DS*DS
      CALL CYL(DS,FH0,FH1,FN1,FN2)
      G1=PI/DS
      G2=PI2-FH1+FN1
      G3=FH0-(FH1+FN1)/DS+FN2
      EFUNC=G1*((4.0*DX2/DS2-1.0)*G2+4.0*DX2*G3/DS)
      IF (NEP.EQ.2) GO TO 2360
      EPS1=EFUNC
2350  NEP=2
      GO TO 2310
2360  EPS2=EFUNC
2390  EP(JJ)=EPS1+EPS2
      EPF=BETA*DELTZ*DELTx(IREF)/(PI4*ER2)
2400  EPS(K)=EPF*(EP(1)+EP(2)+EP(3)+EP(4))/4.0
2500  CONTINUE
2600  CONTINUE
      SYMF=1.0
      IF (ILIFT.EQ.0) SYMF=2.0
      GO TO (3000,5000,5100), METHOD
C
C   THE NEWTON-RAPHSON METHOD
C
3000  CONTINUE
3020  DO 3200 I=1,NS
      KKK=(I-1)*NS+I
      SUM=0.0
      DO 3120 J=1,NS
      K=(J-1)*NS+I
      IJ=IABS(I-J)
      IF (IJ.EQ.0) GO TO 3120
      SUM=SUM+SYMF*EPS(K)*U(J)*U(J)
3120  CONTINUE
      SUM=SUM+SYMF*EPS(KKK)*UL(I)*UL(I)
      ADD=0.0
      IF (ILIFT.EQ.0) GO TO 3150
      II=I+LXZ
      IF (I.GT.LXZ) II=I-LXZ
3140  ADD=0.5*(U(I)-UL(I)-U(II)+UL(II))
3150  FINT(I)=U(I)-0.5*U(I)*U(I)-UL(I)+SUM+ADD
      KK=NS2+I
      A(KK)=FINT(I)
3200  CONTINUE
      DO 3250 I=1,NS
      DO 3240 J=1,NS
      K=(J-1)*NS+I
      IJ=IABS(I-J)
      IF (IJ.EQ.0) GO TO 3230
      ADD=0.0

```

```

      IF (IJ.EQ.LXZ) ADD=-0.5
      A(K)=2.0*SYMF*U(J)*EPS(K)+ADD
      GO TO 3240
3230 ADD=0.5
      IF (ILIFT.EQ.0) ADD=0.0
      A(K)=1.0-U(J)+ADD
3240 CONTINUE
3250 CONTINUE
      CALL GAUSS(A,NS,IT)
      DO 3310 I=1,NS
      KK=NS2+I
      U(I)=U(I)-A(KK)
3310 CONTINUE
      NIT=NIT+1
      DO 3320 I=1,NS
      UTEST=ABS(U(I)-UOL(I))
      IF (UTEST.GT.ACC) GO TO 3330
3320 CONTINUE
      GO TO 5800
3330 CONTINUE
      IF (NIT.GE.15) GO TO 5800
      DO 3340 I=1,NS
      UOL(I)=U(I)
3340 CONTINUE
      GO TO 3020
C
C   THE METHOD OF PARAMETRIC DIFFERENTIATION
C
5000 PRMT(1)=0.0
      PRMT(2)=1.0
      PRMT(3)=0.2
      PRMT(4)=0.1
      NPD=0
      DO 5010 I=1,NS
      Y(I)=UL(I)
5010 DERY(I)=1.0/FLOAT(NS)
      CALL HPCG(PRMT,NS,IHLF,AUX)
      DO 5020 I=1,NS
      UOL(I)=Y(I)
5020 U(I)=Y(I)
      GO TO 5800
C
C   THE METHOD OF STEEPEST DESCENT
C
5100 DO 5050 NZ=1,LZ
      IFG1=0
      IFG2=0
      DO 5040 NX=1,LX
      K=(NZ-1)*LX+NX
      KK=K+LXZ
      IF (IFG1.EQ.1) GO TO 5030
      IF (U(K).GT.OCR) IFG1=1
      XSU(NZ)=XW(K)+DELTX(NZ)/2.0
5030 CONTINUE
      IF (ILIFT.EQ.0) GO TO 5040
      IF (IFG2.EQ.1) GO TO 5040
      IF (U(KK).GT.OCR) IFG2=1
      XSL(NZ)=XW(KK)+DELTX(NZ)/2.0
5040 CONTINUE

```

```

5050 CONTINUE
DO 4850 NZ=1,LZ
  IFG1=0
  IFG2=0
  DO 4840 NX=1,LX
    NNX=LX-NX+1
    K=(NZ-1)*LX+NNX
    KK=K+LXZ
    IF (IFG1.EQ.1) GO TO 4830
    IF (U(K).GT.OCR) IFG1=1
    XXU(NZ)=XW(K)+0.5*DELTX(NZ)
4830 CONTINUE
    IF (ILIFT.EQ.0) GO TO 4840
    IF (IFG2.EQ.1) GO TO 4840
    IF (U(KK).GT.OCR) IFG2=1
    XXL(NZ)=XW(KK)+0.5*DELTX(NZ)
4840 CONTINUE
4850 CONTINUE
WRITE (6,SHOCK)

```

C

```

  NH=NS/2
  LMIN=1
  LMAX=NS
  IMIN=1
  IMAX=NS
  NIT=0
4000 AC=0.0
  IF (ILIFT.EQ.0) GO TO 4020
  SUM1=0.0
  SUM2=0.0
  DO 4010 I=1,NH
    II=I+NH
    SUM1=SUM1+U(I)-UL(I)-U(II)+UL(II)
    SUM2=SUM2+UL(I)*UL(I)-UL(II)*UL(II)
4010 CONTINUE
  AC=0.5*SUM1/SUM2
4020 CONTINUE
  DO 4173 I=LMIN,LMAX
    SUM=0.0
    SUM1=0.0
    KK=(I-1)*NS+I
    DO 4151 J=IMIN,IMAX
      K=(J-1)*NS+I
      IF (J.EQ.I) GO TO 4151
      SUM1=SUM1+SYMF*EPS(K)*U(J)*U(J)
4151 CONTINUE
      SUM1=SUM1+SYMF*EPS(KK)*UL(I)*UL(I)
      CC=0.0
      IF (ILIFT.EQ.0) GO TO 4152
      II=I+NH
      IF (I.GT.NH) II=I-NH
      CC=UL(I)*UL(I)-UL(II)*UL(II)
4152 TER=1.0+2.0*(SUM1-UL(I)+CC*AC)
      TER=SQRT(ABS(TER))
      ZGN=1.0
      IREF=IDZ(I)
      IF (I.GT.LXZ) GO TO 4155
      IF (XW(I).GT.XSU(IREF).AND.XW(I).LT.XXU(IREF)) ZGN=-1.0
      GO TO 4160

```

```

4155 CONTINUE
      IF (XW(I).GT.XSL(IREF).AND.XW(I).LT.XXL(IREF)) ZGN=-1.0
4160 SUM=2.0*(U(I)-1.0+ZGN*TER)
      SUMM=0.0
      DO 4172 J=LMIN,LMAX
        IF (J.EQ.I) GO TO 4172
        KK=(J-1)*NS+J
        KKK=(I-1)*NS+J
        SUM2=0.0
        DO 4165 MJ=IMIN,IMAX
          K=(MJ-1)*NS+J
          IF (MJ.EQ.J) GO TO 4165
          SUM2=SUM2+SYMF*EPS(K)*U(MJ)*U(MJ)
4165 CONTINUE
          SUM2=SUM2+SYMF*EPS(KK)*UL(J)*UL(J)
          CC=0.0
          IF (ILIFT.EQ.0) GO TO 4166
          JJ=J+NH
          IF (J.GT.NH) JJ=J-NH
          CC=UL(J)*UL(J)-UL(JJ)*UL(JJ)
4166 TER=1.0+2.0*(SUM2-UL(J)+CC*AC)
          TER=SQRT(ABS(TER))
          ZKN=1.0
          JREF=IDZ(J)
          IF (J.GT.LXZ) GO TO 4170
          IF (XW(J).GT.XSU(JREF).AND.XW(J).LT.XXU(JREF)) ZKN=-1.0
          GO TO 4171
4170 CONTINUE
          IF (XW(J).GT.XSL(JREF).AND.XW(J).LT.XXL(JREF)) ZKN=-1.0
4171 SU4=2.0*(U(J)-1.0+ZKN*TER)
          SU5=ZKN/(2.0*TER)
          SU6=4.0*U(I)*SYMF*EPS(KKK)
          SUMM=SUMM+SU4*SU5*SU6
4172 CONTINUE
4173 DFU(I)=SUM+SUMM
C
      SUM2=0.0
      DO 4189 I=LMIN,LMAX
        KK=(I-1)*NS+I
        SUM1=0.0
        DO 4178 J=IMIN,IMAX
          K=(J-1)*NS+I
          IF (J.EQ.I) GO TO 4178
          SUM1=SUM1+SYMF*EPS(K)*U(J)*U(J)
4178 CONTINUE
          SUM1=SUM1+SYMF*EPS(KK)*UL(I)*UL(I)
          CC=0.0
          IF (ILIFT.EQ.0) GO TO 4179
          II=I+NH
          IF (I.GT.NH) II=I-NH
          CC=UL(I)*UL(I)-UL(II)*UL(II)
4179 TER=1.0+2.0*(SUM1-UL(I)+CC*AC)
          TER=SQRT(ABS(TER))
          ZGN=1.0
          IREF=IDZ(I)
          IF (I.GT.LXZ) GO TO 4180
          IF (XW(I).GT.XSU(IREF).AND.XW(I).LT.XXU(IREF)) ZGN=-1.0
          GO TO 4181
4180 CONTINUE

```

```

      IF (XW(I).GT.XSL(IREF).AND.XW(I).LT.XXL(IREF)) ZGN=-1.0
4181 TER1=U(I)-1.0+ZGN*TER
4189 SUM2=SUM2+TER1*TER1
C
      FFF=SUM2
      SUM=0.0
      DO 4190 I=LMIN,LMAX
4190 SUM=SUM+DFU(I)*DFU(I)
      ELL=FFF/SUM
      DO 4200 I=LMIN,LMAX
      U(I)=U(I)-ELL*DFU(I)
4200 CONTINUE
      NIT=NIT+1
      DO 4700 I=1,NS
      UTEST=ABS(U(I)-UOL(I))
      IF (UTEST.GT.ACC) GO TO 4710
4700 CONTINUE
      GO TO 5800
4710 CONTINUE
      IF (NIT.GE.25) GO TO 5800
      DO 4720 I=1,NS
4720 UOL(I)=U(I)
      GO TO 4000
C
C      WRITE OUT THE SOLUTION FOR THE WING
C
5800 NWR=1
      IF (NRUN.GT.0) GO TO 6010
      GO TO (5810,5890,5910), IWR
5810 WRITE (6,5820)
5820 FORMAT (1H0,3X,11H PLANFORM- )
      WRITE (6,5830)
5830 FORMAT (1H ,3X,10H PROFILE- )
      GO TO 5950
5890 WRITE (6,5900)
5900 FORMAT (1H0,3X,20H PLANFORM- RECTANGLE )
      GO TO 5930
5910 WRITE (6,5920)
5920 FORMAT (1H0,3X,19H PLANFORM- TRIANGLE )
5930 WRITE (6,5940)
5940 FORMAT (1H ,3X,23H PROFILE- PARABOLIC ARC )
5950 WRITE (6,5960) TAU,SEMI,AR
5960 FORMAT (1H ,3X,
      1 18H THICKNESS RATIO = F12.5 / 4X,
      2 17H SEMISPAN/CHORD = F13.5 / 4X,
      3 15H ASPECT RATIO = F15.5)
6010 WRITE (6 ,6020) AM,BETA,XM,CPST,EMR
6020 FORMAT (1H0,53X,26H THE SOLUTION FOR THE WING // 4X,
      1 15H MACH NUMBER = F15.5 / 4X,
      2 8H BETA = F22.5 / 4X,
      3 22H MACH NUMBER (STAR) = F8.5 /4X,
      4 13H CP (STAR) = F17.5 / 4X,
      5 6H XI = F24.5 // 41X,
      6 52H VELOCITY AND PRESSURE DISTRIBUTION ON UPPER SURFACE /)
6025 WRITE (6,6030)
6030 FORMAT (1H0,4X,
      1 2H I,5X,8H X/CHORD,7X,2H U,11X,4H U+1,7X,7H U(RED),9X,2HCP,8X,
      2 8H CP(RED),6X,5H P/P0,10X,2H M,8X,8H M(STAR) /)
      DO 6200 J=1,LZ

```

```

      NN=(J-1)*LX+1
      ZS=ZW(NN)/SEMI
      WRITE (6,6040) ZW(NN),ZS
6040  FORMAT (1H0,3X,29H SPANWISE STATION, Z/CHORD = F8.5,2X,
1 15H (Z/SEMI SPAN = F8.5,1H) /)
      DO 6200 I=1,LX
      K=(J-1)*LX+I
      KUK=K+LXZ
      IF (NWR.EQ.2) K=KUK
      UWR=U(K)
      UW=UWR/RU
      WW=UPG(K)
      WWR=WW*RU
      RTT=UW+1.0
      WTT=WW+1.0
      CALL VECON(UW,CP,RTM,RMS,PPP)
      CALL VECON(WW,WP,WTM,WMS,WPP)
      CPR=AMT*CP
      WPR=AMT*WP
      XLCHL=(FLOAT(I)-0.5)/FLOAT(LX)
      WRITE (6,6050) 1,XLCHL,UW,RTT,UWR,CP,CPR,PPP,RTM,RMS
6050  FORMAT (5X,I2,1X,9(1X,E12.5))
      WRITE (6,6060) WW,WTT,WWR,WP,WPR,WPP,WTM,WMS
6060  FORMAT (21X,8(1X,E12.5))
6200  CONTINUE
      NWR=NWR+1
      IF (NWR.GE.3) GO TO 8000
      IF (ILIFT.EQ.0) GO TO 8000
      WRITE (6,6300)
6300  FORMAT (// 39X,
1 53H VELOCITY AND PRESSURE DISTRIBUTION ON LOWER SURFACE /)
      GO TO 6025
C
8000  WRITE (6,8030)
8030  FORMAT (1H0,48X,
1 36H THE LINEARIZED SOLUTION IS BASED ON )
      IF (LWING.GT.0) GO TO 8050
      WRITE (6,8040)
8040  FORMAT (1H ,50X,32H CARMICHAEL'S NUMERICAL ANALYSIS /)
      GO TO 8070
8050  WRITE (6,8060)
8060  FORMAT (1H ,52X,28H HOLME'S ANALYTICAL ANALYSIS /)
8070  CONTINUE
      WRITE (6,9000)
9000  FORMAT (1H0,46X,
1 41H THE NONLINEAR SOLUTION WAS OBTAINED WITH)
      GO TO (9010,9030,9050), METHOD
9010  WRITE (6,9020)
9020  FORMAT (1H ,56X,20H THE NEWTON'S METHOD)
      GO TO 9070
9030  WRITE (6,9040) DP
9040  FORMAT (1H ,46X,41H THE METHOD OF PARAMETRIC DIFFERENTIATION /
1 55X,19H AT THE VALUE DP = F6.4)
      GO TO 9085
9050  WRITE (6,9060)
9060  FORMAT (1H ,47X,31H THE METHOD OF STEEPEST DESCENT)
9070  WRITE (6,9080) NIT
9080  FORMAT (1H ,56X, 7H AFTER I2,11H ITERATIONS )
9085  CONTINUE

```


C

```

NRUN=NRUN+1
IF (NRUN.GT.NDELM) GO TO 9090
AM=AM+DELM
GO TO 215
9090 IWIN=9
GO TO 10
9999 CONTINUE
STOP
END

```

C

```

SUBROUTINE SICI(S,SINT,CINT)
S2=S*S
S5=S2
S4=S2*S2
S6=S4*S2
S8=S6*S2
IF (S.GT.1.0) GO TO 20
S3=S5*S
S5=S3*S5
S7=S5*S5
S9=S7*S5
S3=S3/18.0
S5=S5/600.0
S7=S7/35280.0
S9=S9/3265920.0
SINT = S-S3+S5-S7+S9
S2=S2/4.0
S4=S4/96.0
S6=S6/4320.0
S8=S8/322560.0
CINT = 0.57721566+ALOG(S)-S2+S4-S6+S8
GO TO 30
20 F1=S8+38.027264*S6+265.187033*S4+335.67732*S2+38.102495
F2=S6+40.021433*S6+332.624911*S4+570.23628*S2+157.105423
F0=F1/(S*F2)
G1=S6+42.242855*S6+302.757865*S4+352.018498*S2+21.821899
G2=S8+48.196927*S6+482.485924*S4+1114.978885*S2+449.690326
G0=G1/(S5*G2)
SINT=1.5707963-F0*COS(S)-G0*SIN(S)
CINT=F0*SIN(S)-G0*COS(S)
30 CONTINUE
RETURN
END

```

C

```

SUBROUTINE CYL(S,FH0,FH1,FN1,FN2)

PI=3.1415926535
PI2=0.63661977
IF (S.LT.4.0) GO TO 40
T1=4.0/S
T2=T1*T1
P0=((((-0.0000037043*T2+0.0000173565)*T2-0.0000487613)*T2
1 +0.00017343)*T2-0.001753062)*T2+0.3989423
Q0=((((-0.0000032312*T2-0.0000142078)*T2+0.0000342468)*T2
1 +0.0000869791)*T2+0.0004564324)*T2-0.01246694
P1=((((-0.0000042414*T2-0.000020092)*T2+0.0000580759)*T2
1 +0.000223203)*T2+0.002921826)*T2+0.3989423
Q1=((((-0.0000036594*T2+0.00001622)*T2-0.0000398708)*T2
1 +0.0001064741)*T2-0.0006390400)*T2+0.03740084
A=2.0/SQRT(S)
B=A*T1
C=S-0.7853982
Y0=A*P0*SIN(C)+B*Q0*COS(C)
Y1=-A*P1*COS(C)+B*Q1*SIN(C)
GO TO 90
40 SS=S/2.
S2=SS*SS
T=ALOG(SS)+.5772156649
SUM=0.0
TERM=T
Y0=T
DO 70 L=1,15
IF (L-1) 50,60,50
50 SUM=SUM+1.0/FLOAT(L-1)
60 FL=FLOAT(L)
TS=T-SUM
TERM=(TERM*(-S2)/FL**2)*(1.0-1.0/(FL*TS))
70 Y0=Y0+TERM
TERM=SS*(T-.5)
SUM=0.0
Y1=TERM
DO 80 L=2,16
SUM=SUM+1./FLOAT(L-1)
FL=FLOAT(L)
FL1=FL-1.
TS=T-SUM
TERM=(TERM*(-S2)/(FL1*FL))*((TS-.5/FL)/(TS+.5/FL1))
80 Y1=Y1+TERM
Y0=PI2*Y0
Y1=-PI2/S+PI2*Y1
90 Y2=2.0*Y1/S-Y0
FN1=Y1
FN2=Y2
IF (S.GT.11.0) GO TO 120
SH=S/2.0
SH2=SH*SH
A0=8.0/(3.0*PI)
B0=4.0/PI
AN=-4.0*SH2*A0/15.0
BN=-4.0*SH2*B0/9.0
SUM=A0+AN

```

```

      ZUM=B0+BN
      DO 110 I=1,16
      FLI=FLOAT(I)
      AN=-SH2/((FLI+2.5)*(FLI+1.5))*AN
      BN=-SH2*BN/(FLI+1.5)**2
      SUM=SUM+AN
110   ZUM=ZUM+BN
      FH0=SH*ZUM
      FH1=SH2*SUM
      GO TO 130
120   S2=S*S
      S3=S2*S
      S4=S2*S2
      S5=S4*S
      S6=S2*S4
      S7=S6*S
      S8=S2*S6
      S9=S6*S
      S10=S2*S8
      S11=S10*S
      S12=S2*S10
      FH0=Y0+PI2*(1.0/S-1.0/S3+9.0/S5-
1 225.0/S7+11025.0/S9-893025.0/S11)
      FH1=FN1+PI2*(1.0+1.0/S2-3.0/S4+45.0/S6-1575.0/S8+
1 99225.0/S10-9823275.0/S12)
130  CONTINUE
      RETURN
      END

```

```

C      FUNCTION ACSH(CA)
      CQ=CA*CA
      IF (CA.GT.1.0) GO TO 40
      C3=CQ*CA
      C5=CQ*C3
      C7=CQ*C5
      C9=CQ*C7
      C11=CQ*C9
      ACSH=CA-C3/6.0+3.0*C5/40.0-15.0*C7/336.0+
1 105.0*C9/3456.0-945.0*C11/42240.0
      GO TO 50
40   C4=CQ*CQ
      C6=CQ*C4
      C8=CQ*C6
      C10=CQ*C8
      ACSH=ALOG(2.*CA)+1.0/(4.0*CQ)-3.0/(32.0*C4)+
1 15.0/(288.0*C6)-105.0/(3072.0*C8)+945.0/(38400.0*C10)
50  CONTINUE
      RETURN
      END

```

C

```
SUBROUTINE GAUSS(A,N,IT)
  DIMENSION A(N,1)
  REAL MAX
  IT=0
  E=1.E-8
  NN=N+1
  DO 12 I=1,N
    IF (I-1) 3,3,1
  1 DO 2 J=I,N
    M=I-1
    DO 2 K=1,M
  2 A(J,I)=A(J,I)-A(J,K)*A(K,I)
  3 KEY=I
    MAX=ABS(A(I,I))
    DO 5 K=I,N
      IF (MAX-ABS(A(K,I))) 4,5,5
  4 KEY=K
    MAX=ABS(A(K,I))
  5 CONTINUE
    IF (MAX-E) 6,7,7
  6 IT=1
    RETURN
  7 DO 8 K=1,NN
    MAX=A(I,K)
    A(I,K)=A(KEY,K)
  8 A(KEY,K)=MAX
    II=I+1
    M=I-1
    DO 12 J=II,NN
      IF (I-1) 9,11,9
  9 DO 10 K=1,M
  10 A(I,J)=A(I,J)-A(I,K)*A(K,J)
  11 A(I,J)=A(I,J)/A(I,I)
  12 CONTINUE
    DO 14 I=2,N
      J=N+1-I
      JJ=J+1
      DO 13 K=JJ,N
  13 A(J,NN)=A(J,NN)-A(J,K)*A(K,NN)
  14 CONTINUE
    RETURN
  END
```

SUBROUTINE HPCG(PRMT,NDIM,IHLF,AUX)

C

```

DIMENSION PRMT(1),AUX(16,1)
COMMON /DY/ DERY(100),Y(100),UL(100),DP
N=1
IHLF=0
X=PRMT(1)
H=PRMT(3)
PRMT(5)=0.
DO 1 I=1,NDIM
  AUX(16,I)=0.
  AUX(15,I)=DERY(I)
1  AUX(1,I)=Y(I)
  IF (H*(PRMT(2)-X)) 3,2,4
2  IHLF=12
  GO TO 4
3  IHLF=13
4  CALL FCT(X,NDIM)
  CALL OUTP(X,NDIM,IHLF)
  IF (PRMT(5)) 6,5,6
5  IF (IHLF) 7,7,6
6  RETURN
7  DO 8 I=1,NDIM
8  AUX(8,I)=DERY(I)
  ISW=1
  GO TO 100
9  X=X-H
  DO 10 I=1,NDIM
10  AUX(2,I)=Y(I)
11  IHLF=IHLF+1
  X=X-H
  DO 12 I=1,NDIM
12  AUX(4,I)=AUX(2,I)
  H=0.5*H
  N=1
  ISW=2
  GO TO 100
13  X=X+H
  CALL FCT(X,NDIM)
  N=2
  DO 14 I=1,NDIM
  AUX(2,I)=Y(I)
14  AUX(9,I)=DERY(I)
  ISW=3
  GO TO 100
15  DELT=0.0
  DO 16 I=1,NDIM
16  DELT=DELT+AUX(15,I)*ABS(Y(I)-AUX(4,I))
  DELT=0.06666667*DELT
  IF (DELT-PRMT(4)) 19,19,17
17  IF (IHLF-10) 11,18,18
18  IHLF=11
  X=X+H
  GO TO 4
19  X=X+H
  CALL FCT(X,NDIM)
  DO 20 I=1,NDIM

```

```

    AUX(3,I)=Y(I)
20  AUX(10,I)=DERY(I)
    N=3
    ISW=4
    GO TO 100
21  N=1
    X=X+H
    CALL FCT(X,NDIM)
    X=PRMT(1)
    DO 22 I=1,NDIM
        AUX(11,I)=DERY(I)
22  Y(I)=AUX(1,I)+H*(0.375*AUX(8,I)+0.7916667*AUX(9,I)
1  -0.2083333*AUX(10,I)+0.0416667*DERY(I))
23  X=X+H
    N=N+1
    CALL FCT(X,NDIM)
    CALL OUTP(X,NDIM,IHLF)
    IF (PRMT(5)) 6,24,6
24  IF (N=4) 25,200,200
25  DO 26 I=1,NDIM
    AUX(N,I)=Y(I)
26  AUX(N+7,I)=DERY(I)
    IF (N=3) 27,29,200
27  DO 28 I=1,NDIM
    DELT=AUX(9,I)+AUX(9,I)
    DELT=DELT+DELT
28  Y(I)=AUX(1,I)+0.3333333*H*(AUX(8,I)+DELT+AUX(10,I))
    GO TO 23
29  DO 30 I=1,NDIM
    DELT=AUX(9,I)+AUX(10,I)
    DELT=DELT+DELT+DELT
30  Y(I)=AUX(1,I)+0.375*H*(AUX(8,I)+DELT+AUX(11,I))
    GO TO 23
100 DO 101 I=1,NDIM
    Z=H*AUX(N+7,I)
    AUX(5,I)=Z
101 Y(I)=AUX(N,I)+0.4*Z
    Z=X+0.4*H
    CALL FCT(Z,NDIM)
    DO 102 I=1,NDIM
    Z=H*DERY(I)
    AUX(6,I)=Z
102 Y(I)=AUX(N,I)+0.2969776*AUX(5,I)+0.1587596*Z
    Z=X+0.4557372*H
    CALL FCT(Z,NDIM)
    DO 103 I=1,NDIM
    Z=H*DERY(I)
    AUX(7,I)=Z
103 Y(I)=AUX(N,I)+0.2181004*AUX(5,I)-3.050965*AUX(6,I)+3.832865*Z
    Z=X+H
    CALL FCT(Z,NDIM)
    DO 104 I=1,NDIM
104 Y(I)=AUX(N,I)+0.1747603*AUX(5,I)-0.5514807*AUX(6,I)
1  +1.205536*AUX(7,I)+0.1711948*DERY(I)
    GO TO (9,13,15,21), ISW
200 ISTEP=3
201 IF (N=8) 204,202,204
202 DO 203 N=2,7
    DO 203 I=1,NDIM

```

```

      AUX(N-1,I)=AUX(N,I)
203  AUX(N+6,I)=AUX(N+7,I)
      N=N+1
204  DO 205 I=1,NDIM
      AUX(N-1,I)=Y(I)
205  AUX(N+6,I)=DERY(I)
      X=X+H
      ISTEP=ISTEP+1
      DO 207 I=1,NDIM
      DELT=AUX(N-4,I)+1.333333*H*(AUX(N+6,I)+AUX(N+6,I)-AUX(N+5,I)+
1  AUX(N+4,I)+AUX(N+4,I))
      Y(I)=DELT-0.9256198*AUX(16,I)
207  AUX(16,I)=DELT
      CALL FCT(X,NDIM)
      DO 208 I=1,NDIM
      DELT=0.125*(9.0*AUX(N-1,I)-AUX(N-3,I)+3.0*H*(DERY(I)+AUX(N+6,I)+
1  AUX(N+6,I)-AUX(N+5,I)))
      AUX(16,I)=AUX(16,I)-DELT
208  Y(I)=DELT+0.0743817*AUX(16,I)
      DELT=0.0
      DO 209 I=1,NDIM
209  DELT=DELT+AUX(15,I)*ABS(AUX(16,I))
      IF (DELT-PRMT(4)) 210,212,212
210  CALL FCT(X,NDIM)
      CALL OUTP(X,NDIM,IHLF)
      DP=X
      IF (PRMT(5)) 212,211,212
211  IF (IHLF-11) 213,212,212
212  RETURN
213  IF (H*(X-PRMT(2))) 214,212,212
214  IF (ABS(X-PRMT(2))-0.1*ABS(H)) 212,201,201
      END

```

SUBROUTINE FCT(X,NDIM)

C

```

COMMON /EX/ EX(50)
COMMON /DY/ DERY(100),Y(100),UL(100),DP
COMMON /PRNDTL/ UPG(100),ILIFT,NRUN,SYMF
COMMON /HOL/ EP(10000),A(10100)
IT=9
NH=NDIM/2
DO 1200 I=1,NDIM
SUM=0.0
DO 1120 J=1,NDIM
K=(J-1)*NDIM+I
IJ=IABS(I-J)
IF (IJ.EQ.0) GO TO 1110
SUM=SUM+SYMF*EP(K)*Y(J)*Y(J)
GO TO 1120
1110 SUM=SUM+SYMF*EP(K)*UL(J)*UL(J)
1120 CONTINUE
KK=NDIM*NDIM+I
IF (ILIFT.EQ.0) GO TO 1130
II=I+NH
IF (I.GT.NH) II=I-NH

```

```

      A(KK)=0.5*(Y(I)*Y(I)-Y(I)+UL(I)+Y(II)-UL(II))-SUM
      GO TO 1200
1130  A(KK) = 0.5*Y(I)*Y(I)-SUM
1200  CONTINUE
      DO 1250 I=1,NDIM
      DO 1240 J=1,NDIM
      K=(J-1)*NDIM+I
      KK=(J-1)*NDIM+I
      IJ=IABS(I-J)
      IF (IJ.EQ.0) GO TO 1230
      ADD=0.0
      IF (IABS(I-J).EQ.NH) ADD=-0.5*X
      A(KK)=2.0*SYMF*X*Y(J)*EP(K)+ADD
      GO TO 1240
1230  A(KK)=1.0-X*(Y(J)-0.5)
1240  CONTINUE
1250  CONTINUE
      CALL GAUSS(A,NDIM,IT)
      DO 1310 I=1,NDIM
      KK=NDIM*NDIM+I
1310  DERY(I)=A(KK)
      RETURN
      END

```

```

SUBROUTINE OUTP(X,NDIM,IHLF)
C
C  COMMON /DY/ DERY(100),Y(100),UL(100),DP
C  OPTION FOR USER TO PRINT OUT INTERMEDIATE SOLUTION
      RETURN
      END

```

```

SUBROUTINE VECON(X,CP,AM,AMS,PR)
C
COMMON /VEL/ MACH,MACHSQ,BETASQ,BETA,AAM1,AAM2,AAM3
REAL MACH,MACHSQ,BETASQ,BETA
X1=X+1.0
X12=X1*X1
CP1=ABS(1.0+AAM2*(1.0-X12))
CP=AAM1*(CP1**3.5-1.0)
AM=MACH*X1/SQRT(ABS(1.0+AAM2*(1.0-X12)))
AMS=AM*SQRT(ABS(1.0/(1.0+0.16666666*(AM*AM-1.0))))
PR=AAM3*CP+1.0
      RETURN
      END

```


SUBROUTINE CARM

```

COMMON /VEL/ MACH,MACHSQ,BETASQ,BETA,AAM1,AAM2,AAM3
COMMON /PARAMS/ NHING,PANELS,SREF,REFMOM,CBAR,SPAN,OC
COMMON /WNGPMS/ ROOT(4),TIP(4),M,N,TYPE,F(101),G(101),P(101),
1 SHEAR(101),ISECT,THICK(5)
COMMON /HEL/ HH(100,100)
REAL MACH,MACHSQ,BETASQ,BETA
INTEGER PANELS,OC,TYPE
LOGICAL SYM
LOGICAL THK
DATA THK/.FALSE./
SYM=.FALSE.
DATA PANELS/0/
DATA HALFPI/1.57079633/,PI/3.14159265/
CALL WNGEOM
NHING=PANELS
IF (THICK(1).GT.0.0.OR.THICK(2).GT.0.0) THK=.TRUE.
CALL EVAL (THK)
N1=NHING+1
CALL INVERT (HH,NHING)
CALL FORCE
PANELS=0
RETURN
END

```

SUBROUTINE WNGEOM

```

COMMON /VEL/ MACH,MACHSQ,BETASQ,BETA,AAM1,AAM2,AAM3
COMMON /PARAMS/ NHING,PANELS,SREF,REFMOM,CBAR,SPAN,OC
COMMON /WNGPMS/ ROOT(4),TIP(4),M,N,TYPE,F(101),G(101),P(101),
1 SHEAR(101),ISECT,THICK(2),PERCHD,ZT,DZDX,CURX
COMMON /PSINGS/ PW(100),ALPHAT(100)
COMMON /PDATA/ XBAR(100),AREA(100),COSTH(100),SINTH(100),SYM(100)
COMMON /SCRAP/ X(100,4),Y(100,2),Z(100,2),XCPT(101),
1 SLOPE(101),CHROOT(202),CHTIP(202),ZUROOT(101),ZUTIP(101),
2 ZLROOT(101),ZLTIP(101),SCF(101)
COMMON /CURV/ CUR(100)
REAL MACH,MACHSQ,BETASQ,BETA
INTEGER PANELS,PANMAX,OC,TYPE
LOGICAL SYM,FIN
DATA PANMAX /100/
HELL=0.000001
IF (M.LE.0.OR.N.LE.0) RETURN
M1=M+1
N1=N+1
F(1)=ROOT(1)
G(1)=TIP(1)
F(N1)=ROOT(2)
G(N1)=TIP(2)
P(1)=ROOT(3)
P(N1)=TIP(3)
SHEAR(1)=ROOT(4)

```

```

SHEAR(M1)=TIP(4)
DR=ROOT(2)-ROOT(1)
DT=TIP(2)-TIP(1)
SSPAN=TIP(3)-ROOT(3)
PAREA=(DR+DT)*SSPAN
SC1=1.
SC2=1.
IF (TYPE.EQ.1) GO TO 20
IF (TYPE.EQ.2) GO TO 10
CALL FILL (F,N)
CALL FILL (G,N)
10 IF (TYPE.EQ.3) GO TO 20
CALL FILL (P,M)
CALL FILL (SHEAR,M)
20 XX=TIP(3)-ROOT(3)
DO 30 I=1,N1
CHROOT(I)=0.
CHTIP(I)=0.
SLOPE(I)=(G(I)-F(I))/XX
30 XCPT(I)=F(I)-ROOT(3)*SLOPE(I)
DO 50 I=1,N
IF (ABS(DR).LT.HELL) GO TO 40
CHROOT(I)=((F(I)+F(I+1))/2.-F(1))/DR
40 IF (ABS(DT).LT.HELL) GO TO 50
CHTIP(I)=((G(I)+G(I+1))/2.-G(1))/DT
50 CONTINUE
IF (DT.GT.0.) GO TO 70
DO 60 I=1,N
60 CHTIP(I)=CHROOT(I)
70 DO 90 J=1,M
Y1=P(J)
Y2=P(J+1)
YPER=((Y1+Y2)/2.-P(1))/(TIP(3)-ROOT(3))
YPER1=1.0-YPER
THCK=THICK(2)*YPER+THICK(1)*YPER1
DO 90 I=1,N
PANELS=PANELS+1
IF (PANELS.GT.PANMAX) GO TO 210
X(PANELS,1)=XCPT(I)+Y1*SLOPE(I)
X(PANELS,2)=XCPT(I)+Y2*SLOPE(I)
X(PANELS,3)=XCPT(I+1)+Y1*SLOPE(I+1)
X(PANELS,4)=XCPT(I+1)+Y2*SLOPE(I+1)
Y(PANELS,1)=Y1
Y(PANELS,2)=Y2
Z(PANELS,1)=SHEAR(J)
Z(PANELS,2)=SHEAR(J+1)
IF (ABS(THCK).LT.HELL) GO TO 80
PERCHD=CHTIP(I)*YPER+CHROOT(I)*YPER1
CALL SECTIN
CUR(PANELS)=THCK*CURX
80 ALPHAT(PANELS)=THCK*DZDX
FIN=.FALSE.
IF (ABS(Y1).LT.HELL.AND.ABS(Y2).LT.HELL) FIN=.TRUE.
90 SYN(PANELS)=.NOT.FIN
C CALCULATIONS OF WING PROFILE
RTMAX=THICK(1)*DR
TIPMAX=THICK(2)*DT
DO 130 I=1,N1
IF (ABS(RTMAX).LT.HELL.OR.ABS(DR).LT.HELL) GO TO 100

```

```

      PERCHD=(F(I)-F(1))/DR
      CALL SECTIN
      ZUROOT(I)=ROOT(4)+ZT*RTMAX
      ZLROOT(I)=ROOT(4)-ZT*RTMAX
      GO TO 110
100  ZUROOT(I)=ROOT(4)
      ZLROOT(I)=ROOT(4)
110  IF (ABS(TIPMAX).LT.HELL.OR.ABS(DT).LT.HELL) GO TO 120
      PERCHD=(G(I)-G(1))/DT
      CALL SECTIN
      ZUTIP(I)=TIP(4)+ZT*TIPMAX
      ZLTIP(I)=TIP(4)-ZT*TIPMAX
      GO TO 130
120  ZUTIP(I)=TIP(4)
      ZLTIP(I)=TIP(4)
130  CONTINUE
210  RETURN
      END

```

SUBROUTINE FILL (AFILL,NFILL)

```

C
      REAL AFILL(1)
      IF (NFILL.LE.1) RETURN
      DEL=(AFILL(NFILL+1)-AFILL(1))/FLOAT(NFILL)
      DO 10 I=2,NFILL
10   AFILL(I)=AFILL(I-1)+DEL
      RETURN
      END

```

SUBROUTINE SECTIN

```

C
      COMMON /WNGPMS/ ROOT(4),TIP(4),M,N,TYPE,F(101),G(101),P(101),
1  SHEAR(101),ISECT,THICK(2),X,Z,DZDX,CURX
      DIMENSION XTAB(19), ZTAB(19), ZPTAB(19), CUTAB(19)
C   THE FOUR DATA TABLES GIVEN BELOW ARE FOR A RAE 101 PROFILE
C   AND THEY REPRESENT 19 CHORDWISE VALUES FOR THE PROFILE
C   X/CHORD STATION TOGETHER WITH THE THICKNESS, SLOPE AND
C   CURVATURE MEASURE RESPECTIVELY AS GIVEN FOR A PROFILE
C   THICKNESS RATIO OF ONE. FOR ARBITRARY PROFILE INPUT THE
C   APPROPRIATE DATA TABLES MUST BE SUPPLIED BY THE USER.
C   IF THE NUMBER OF CHORDWISE STATIONS USED IS DIFFERENT
C   FROM 19, THE DIMENSION, STATEMENT ABOVE AND THE THREE
C   SEQUENTIAL CALLING STATEMENTS STARTING WITH STATEMENT 100
C   MUST ALSO BE CHANGED ACCORDINGLY.
      DATA XTAB /0.,.005,.0125,.0250,.05,.075,.1,.15,.2,.25,.3,.4,
1  .5,.6,.7,.8,.9,.95,1./
      DATA ZTAB /0.,.0871,.1369,.1917,.2659,.3191,.3607,.4220,.4630,
1  .4885,.4997,.4801,.4267,.3531,.2681,.1789,.0894,.0447,0./
      DATA ZPTAB /39.05,8.685,5.265,3.709,2.462,1.863,1.434,1.001,
1  1.6575,.3675,.0640,-.3938,-.6508,-.8058,-.8823,-.8943,-.8943,

```

```

2-.6945,-.8944/
  DATA CUTAB1 /-7000.0,-420.0,-120.0,-83.1,-32.6,-19.601,-12.35,
1 -8.102,-6.915,-5.92,-4.988,-3.384,-2.117,-1.173,-0.602,
2 -0.331,-0.182,-0.150,-0.125 /
  IF (X.LT.0..OR.X.GT.1.) GO TO 130
  IF (ISECT.EQ.0) GO TO 100
  GO TO (10,30,50), ISECT
10 Z=2.*X*(1.-X)
  DZDX=2.*(1.-2.*X)
  CURX=-4.0
  RETURN
30 IF (X.GT..5) GO TO 40
  Z=X
  DZDX=1.
  CURX=-4.0
  RETURN
40 Z=1.-X
  DZDX=-1.
  CURX=-4.0
  RETURN
50 CONTINUE
  IF (X.GT.0.0) GO TO 60
  Z=0.0
  DZDX=100.0
  CURX=-5.0
  RETURN
60 SX=SQRT(X)
  X2=X*X
  Z=5.0*(0.2969*SX-0.126*X-0.3516*X2+
1 0.2643*X2*X-0.1015*X2*X2)
  DZDX=5.0*(0.14845/SX-0.126-0.7032*X+
1 0.8529*X2-0.406*X2*X)
  CURX=-5.0*(0.074225/(SX*X)+0.7032-
1 1.7058*X+1.218*X2)
  RETURN
100 CALL TAIN (XTAB,ZTAB1,X,Z,19,2)
  CALL TAIN (XTAB,ZPTAB1,X,DZDX,19,2)
  CALL TAIN (XTAB,CUTAB1,X,CURX,19,2)
  RETURN
130 Z=0.
  DZDX=0.
  CURX=0.0
  RETURN
END

```

C SUBROUTINE TAIN (XTAB,FTAB,X,FX,N,K)

```

REAL XTAB(1),FTAB(1),C(10),T(10)
DO 10 I=1,N
  IF (X.GT.XTAB(I)) GO TO 10
  J=I
  GO TO 20
10 CONTINUE
  J=N
20 J=J-(K+1)/2
  IF (J.LE.0) J=1
30 M=J+K
  IF (M.LE.N) GO TO 40
  J=J-1
  GO TO 30
40 KP1=K+1
  GO TO 50
  ENTRY TNT
50 DO 60 L=1,KP1
  C(L)=X-XTAB(J)
  T(L)=FTAB(J)
60 J=J+1
  DO 80 J=1,K
  I=J+1
70 T(I)=(C(J)*T(I)-C(I)*T(J))/(C(J)-C(I))
  I=I+1
  IF (I.LE.KP1) GO TO 70
80 CONTINUE
  FX=T(KP1)
  RETURN
  END

```

C SUBROUTINE EVAL (THICK)

```

COMMON /VEL/ MACH,MACHSQ,BETASQ,BETA,AAM1,AAM2,AAM3
COMMON /PARAMS/ NWIN,PANELS,SREF,REFMOM,CBAR,SPAN,OC
COMMON /PDATA/ XBARS(100),AREA(100),COSTHS(100),SINTHS(100),
1 SYN(100)
COMMON /COMPS/ XPRIME,YPRIME,ZPRIME,U,V,W,B,BTERM,EPS,SUB,BPOS,XPM
1T
COMMON /SCRAP/ X(100,4),Y(100,2),Z(100,2),XCS(100),YCS(100),
1 ZCS(100),A(100),XBAR(100),XC(100),YC(100),ZC(100),
2 SINTH(100),COSTH(100),UNT(100)
COMMON /HEL/ HH(100,1,0)
COMMON /HOL/ HH1(100,100),HH2(100,100)
COMMON/XZw/XV(100),ZV(100)
REAL MACH,MACHSQ,BETASQ,BETA
REAL LE
INTEGER PANELS,OC
LOGICAL SUB,BPOS,BNEG,B1NEG,B2NEG,DIAG,WING,THICK,TWING
LOGICAL SYN
DATA PI1/.3183099/,PI2/.1591549/
DATA PI4/7.957747163E-2/,PI6/3.978673581E-2/
USELF=0.0

```

```

STOTAL=0.
BNEG=.FALSE.
DO 10 I=1,PANELS
CR=X(I,3)-X(I,1)
DELY=Y(I,2)-Y(I,1)
DELZ=Z(I,2)-Z(I,1)
SPN=SQRT(DELY*DELY+DELZ*DELZ)
CT=X(I,4)-X(I,2)
AREA(I)=(CR+CT)*SPN/2.
STOTAL=STOTAL+AREA(I)
YPER=(1.+CT/(CR+CT))/3.
YPER1=1.-YPER
LE=YPER*X(I,2)+YPER1*X(I,1)
TE=YPER*X(I,4)+YPER1*X(I,3)
XCS(I)=.95*TE+.05*LE
XC(I)=XCS(I)/BETA
XBARS(I)=(LE+TE)/2.
XV(I)=XBARS(I)
XBAR(I)=XBARS(I)/BETA
YCS(I)=Y(I,1)*YPER1+Y(I,2)*YPER
ZV(I)=YCS(I)
YC(I)=YCS(I)
ZCS(I)=Z(I,1)*YPER1+Z(I,2)*YPER
ZC(I)=ZCS(I)
SINTHS(I)=DELZ/SPN
SINTH(I)=SINTHS(I)
COSTHS(I)=DELY/SPN
10 COSTH(I)=COSTHS(I)
STCTAL=STOTAL*2.
IF (SREF.LT.0.) SREF=STOTAL
IF (CBAR.LT.0.) CBAR=SQRT(SREF)
EPS=AMAX1(SPAN,CBAR)/10000.
IF (BETASQ) 40,40,30
30 SUB=.TRUE.
XBTERM=1.
UCON=PI8
UTCON=PI2/BETA
VWCON=BETA*PI8
VWTCUN=PI2
GO TO 50
40 SUB=.FALSE.
XBTERM=-1.
UCON=PI4
UTCON=PI1/BETA
VWCON=BETA*PI4
VWTCUN=PI1
50 DO 260 I=1,PANELS
COST=COSTH(I)
SINT=SINTH(I)
WING=.TRUE.
TWING=THICK.AND.WING
X1=X(I,1)/BETA
X2=X(I,2)/BETA
X3=X(I,3)/BETA
X4=X(I,4)/BETA
Y1=Y(I,1)
Y2=Y(I,2)
Z1=Z(I,1)
Z2=Z(I,2)

```

```

DELTAY=(Y2-Y1)*COST+(Z2-Z1)*SINT
B1=(X2-X1)/DELTAY
B2=(X4-X3)/DELTAY
B1NEG=B1.LT.0.
B2NEG=B2.LT.0.
B1=ABS(B1)
B2=ABS(B2)
BTERM1=SQRT(ABS(B1*B1+XBTERM))
BTERM2=SQRT(ABS(B2*B2+XBTERM))
DO 250 J=1,PANELS
DIAG=1.EG.J.AND.WING
XW=SINT*COSTH(J)
XX=COST*SINTH(J)
XY=COST*COSSH(J)
XZ=SINT*SINTH(J)
SINTR=XW-XX
COSTR=XY+XZ
SINTL=XW+XX
COSTL=XY-XZ
C BEGIN THE EIGHT SEPARATE SET-UPS FOR COMP AND TCOMP
BPOS=.NOT.B1NEG
B=B1
BTERM=BTERM1
C SETUP 1
DELTAY=YC(J)-Y1
DELTAZ=ZC(J)-Z1
XPRIME=XC(J)-X1
YPRIME=DELTAY*COST+DELTAZ*SINT
YP3=YPRIME
IF (B1NEG) YPRIME=-YPRIME
ZPRIME=DELTAY*COST-DELTAZ*SINT
ZP3=ZPRIME
CALL COMP
AAVR=V
AAWR=W
IF (DIAG) GO TO 60
UU=U
GO TO 70
60 UU=0.
70 IF (.NOT.TWING) GO TO 90
XPMT=XBAR(J)-X1
CALL TCOMP
UT=U
IF (DIAG) GO TO 80
AAVRT=V
AAWRT=W
GO TO 90
80 AAVRT=0.
AAWRT=0.
ABN=V
C SETUP 2
90 DELTAY=-YC(J)-Y1
YPRIME=DELTAY*COST+DELTAZ*SINT
YP3N=YPRIME
IF (B1NEG) YPRIME=-YPRIME
ZPRIME=DELTAY*COST-DELTAZ*SINT
ZP3N=ZPRIME
CALL COMP
AAVL=V

```

```

AAWL=W
UU=UU+U
IF (.NOT.TWING) GO TO 100
CALL TCOMP
UT=UT+U
AAVLT=V
AAWLT=W
C
100 SETUP 3
DELTAY=YC(J)-Y2
DELTAZ=ZC(J)-Z2
XPRIME=XC(J)-X2
YPRIME=DELTAY*COST+DELTAZ*SINT
YP4=YPRIME
IF (B1NEG) YPRIME=-YPRIME
ZPRIME=DELTAY*COST-DELTAZ*SINT
ZP4=ZPRIME
CALL COMP
AAVR=AAVR-V
AAWR=AAWR-W
IF (DIAG) GO TO 110
UU=UU-U
110 IF (.NOT.TWING) GO TO 130
XPMT=XBAR(J)-X2
CALL TCOMP
UT=UT-U
IF (DIAG) GO TO 120
AAVRT=AAVRT-V
AAWRT=AAWRT-W
GO TO 130
120 ABN=ABN-V
C
130 SETUP 4
DELTAY=-YC(J)-Y2
YPRIME=DELTAY*COST+DELTAZ*SINT
YP4N=YPRIME
IF (B1NEG) YPRIME=-YPRIME
ZPRIME=DELTAY*COST-DELTAZ*SINT
ZP4N=ZPRIME
CALL COMP
AAVL=AAVL-V
AAWL=AAWL-W
UU=UU-U
IF (.NOT.TWING) GO TO 140
CALL TCOMP
UT=UT-U
AAVLT=AAVLT-V
AAWLT=AAWLT-W
140 BPOS=.NOT.B2NEG
B=B2
BTERM=BTERM2
C
SETUP 5
XPRIME=XC(J)-X3
YPRIME=YP3
IF (B2NEG) YPRIME=-YPRIME
ZPRIME=ZP3
CALL COMP
AAVR=AAVR-V
AAWR=AAWR-W
IF (DIAG) GO TO 150
UU=UU-U

```



```

150 IF (.NOT.TWING) GO TO 170
    XPMT=XBAR(J)-X3
    CALL TCOMP
    UT=UT-U
    IF (DIAG) GO TO 160
    AAVRT=AAVRT-V
    AAWRT=AAWRT-W
    GO TO 170
160 ABN=ABN-V
C   SETUP 6
170 YPRIME=YP3N
    IF (B2NEG) YPRIME=-YPRIME
    ZPRIME=ZP3N
    CALL COMP
    AAVL=AAVL-V
    AAWL=AAWL-W
    UU=UU-U
    IF (.NOT.TWING) GO TO 180
    CALL TCOMP
    UT=UT-U
    AAVLT=AAVLT-V
    AAWLT=AAWLT-W
C   SETUP 7
180 YPRIME=YP4
    XPRIME=XC(J)-X4
    IF (B2NEG) YPRIME=-YPRIME
    ZPRIME=ZP4
    CALL COMP
    AAVR=AAVR+V
    AAWR=AAWR+W
    IF (DIAG) GO TO 190
    UU=UU+U
190 IF (.NOT.TWING) GO TO 210
    XPNT=XBAR(J)-X4
    CALL TCOMP
    UT=UT+U
    IF (DIAG) GO TO 200
    AAVRT=AAVRT+V
    AAWRT=AAWRT+W
    GO TO 210
200 ABN=ABN+V
C   SETUP 8
210 YPRIME=YP4N
    IF (B2NEG) YPRIME=-YPRIME
    ZPRIME=ZP4N
    CALL COMP
    AAVL=AAVL+V
    AAWL=AAWL+W
    UU=UU+U
    IF (.NOT.TWING) GO TO 220
    CALL TCOMP
    UT=UT+U
    AAVLT=AAVLT+V
    AAWLT=AAWLT+W
220 CONTINUE
    A(J)=(AAVR*SINTR+AAVL*SINTL+AAWR*COSTR+AAWL*COSTL)*VWCON
    UPVAL=UU*UCON
    IF (.NOT.TWING) GO TO 240
    UWT(J)=UT*UTCON

```

```

      GO TO 245
240  UWT(J)=0.
245  HH(J,I)=A(J)
      HH1(1,J)=UWPVAL
      HH2(1,J)=UWT(J)
250  CONTINUE
260  CONTINUE
      IF (.NOT.THICK) WRITE (6,370)
      RETURN
370  FORMAT (49HOWING THICKNESS MATRIX CALCULATION WAS SUPPRESSED)
      END

```

C

```

SUBROUTINE COMP
COMMON /COMPS/ X,Y,Z,U,V,W,B,BTERM,EPS,SUB,BPOS,XT
LOGICAL SUB,BPOS
DATA PI/3.14159265/,HALFPI/1.57079633/
DATA PI32/4.712389/,ZERO/0./,ONE/1./
D2=0.0
10  CONTINUE
      IF (ABS(B).LT.1.0E-8) GO TO 150
      X2=X*X
      Y2=Y*Y
      IF (ABS(Z).LT.EPS) GO TO 120
      Z2=Z*Z
      R2=Y2+Z2
      BR2=B*R2
      D=SQRT(X2+R2)
      A=X-B*Y
      RPRIME=SQRT(A*A+BTERM*BTERM*Z2)
      F6=(X+D)/R2
      F2=ALOG(ABS((B*X+Y+BTERM*D)/RPRIME))
      U=ATAN2(Z+D,BR2-X*Y)+ATAN(Y/Z)
      V=Z*F6-B*U
      W=BTERM*F2-Y*F6-B*ALOG(ABS((X+D)*RPRIME/BR2))
      GO TO 190
120  A=X-B*Y
      AA=ABS(A)
      D=SQRT(X2+Y2)
      F2=0.0
      IF (ABS(B*X+Y+BTERM*D).LT.1.0E-8) GO TO 121
      F2=ALOG(ABS((B*X+Y+BTERM*D)/AA))
121  CONTINUE
      U=HALFPI
      IF (A.LT.ZERO) GO TO 140
      IF (Y.GT.ZERO) GO TO 130
      U=-HALFPI
      GO TO 140
130  U=PI32
140  V=-B*U
      W=BTERM*F2-(X+D)/Y-B*ALOG(ABS((X+D)*AA/B/Y2))
      GO TO 190
150  IF (ABS(Z).LT.EPS) GO TO 160
      X2=X*X
      Y2=Y*Y
      Z2=Z*Z

```

```

R2=Y2+Z2
D=SQRT(X2+R2)
U=ATAN2(Z*D,-X*Y)+ATAN(Y/Z)
F6=(X+D)/R2
V=Z*F6
W=ALOG(ABS((Y+D)/SQRT(X2+Z2)))-Y*F6
GO TO 190
160 D=SQRT(X*X+Y*Y)
U=HALFPI
IF (X.LT.ZERO) GO TO 180
IF (Y.GT.ZERO) GO TO 170
U=-HALFPI
GO TO 180
170 U=PI/2
180 V=ZERO
W=ALOG(ABS((Y+D)/ABS(X)))-(X+D)/Y
185 CONTINUE
190 IF (BPOS) RETURN
U=-U
W=-W
RETURN
ENTRY TCOMP
X2=XT*XT
Y2=Y*Y
A=XT-B*Y
IF (ABS(Z).LT.EPS) GO TO 270
Z2=Z*Z
R2=Y2+Z2
R=SQRT(R2)
D=SQRT(X2+R2)
RPRIME=SQRT(A*A+BTERM*BTERM*Z2)
F2=ALOG(ABS((B*XT+Y+BTERM*D)/RPRIME))/BTERM
U=-F2
V=B*F2-ALOG(ABS((XT+D)/R))
W=ATAN2(Z*D,B*R2-XT*Y)
RETURN
270 D=SQRT(X2+Y2)
F2=0.0
IF (ABS(B*XT+Y+BTERM*D).LT.1.0E-8) GO TO 271
F2=ALOG(ABS((B*XT+Y+BTERM*D)/ABS(A)))/BTERM
271 CONTINUE
W=ZERO
IF (A*Y.GT.ZERO) W=PI
U=-F2
V=B*F2-ALOG(ABS((XT+D)/ABS(Y)))
RETURN
END

```

```

C      SUBROUTINE INVERT (AA,NN)
C
      REAL AA(100,100)
      NNN=NN
      DO 30 I=1,NN
      ITTEST=I
      PIVOT=AA(I,I)
      AA(I,I)=1.
      DO 10 L=1,NN
      LTTEST=L
      ATTEST=AA(I,L)
10  AA(I,L)=AA(I,L)/PIVOT
      DO 30 M=1,NN
      MTTEST=M
      IF (M.EQ.I) GO TO 30
      TT=AA(M,I)
      AA(M,I)=0.
      DO 20 L=1,NN
20  AA(M,L)=AA(M,L)-AA(I,L)*TT
30  CONTINUE
      RETURN
      END

```

```

C      SUBROUTINE FORCE
C
C      THIS SUBROUTINE INCLUDES THE NAMELIST /LIFT/
C      WHICH SPECIFIES THE GLOBAL ANGLE OF ATTACK (IN DEGREES)
C      OF THE WING PLANFORM AND THE CAMBER OR LOCAL ANGLE
C      OF ATTACK (IN RADIANS) OF EACH WING PANEL.
C
      COMMON /VEL/ MACH,MACHSQ,BETASQ,BETA,AAM1,AAM2,AAM3
      COMMON /PARAMS/ NWIN,PANELS,SREF,REFMOM,CBAR,SPAN,OC
      COMMON /PSINGS/ PW(100),ALPHAT(100)
      COMMON /PDATA/ XBAR(100),AREA(100),COSTH(100),SINTH(100),SYM(100)
      COMMON /SCRAP/ UW(100),DUMMY(200),CAMBER(100),ALPHA(100),CP(100),
1  A(100),B(100),CPU(100),CPL(100),DELC(100),UWT(100)
      COMMON /HEL/ HH(100,100)
      COMMON /HOL/ HH1(100,100),HH2(100,100)
      COMMON /PRNDTL/ UPG(100),ILIFT,NRUN,SYMF
      REAL MACH,MACHSQ,BETASQ,BETA
      INTEGER PANELS, OC
      DATA RAD/57.2957795/,ALFA/0.0/
      NAMELIST /LIFT/ ILIFT,CAMBER,ALFA.
      IF (NRUN.GT.0) GO TO 50
      READ (5,LIFT)
      WRITE (6,LIFT)
50  ANGLE=ALFA/RAD
      ARADEG=ALFA
      WRITE (6,200) ARADEG
      IF (NWIN.EQ.0) GO TO 80
      DO 60 I=1,NWIN
60  PW(I)=0.
      DO 70 J=1,NWIN

```

```

      AXX=-CAMBER(J)-ANGLE*COSTH(J)
      DO 70 I=1,NWING
70    PW(I)=PW(I)+HH(I,J)*AXX
80    CONTINUE
      DO 90 I=1,PANELS
        UWT(I)=0.0
90    UW(I)=0.0
        DO 100 J=1,NWING
          PWTJ=ALPHAT(J)
          CLJ=PW(J)
          DO 100 I=1,PANELS
            UWT(I)=UWT(I)+HH2(J,I)*PWTJ
            UW(I)=UW(I)+HH1(J,I)*CLJ
100   CONTINUE
          DO 140 I=1,NWING
            UX=UW(I)+UWT(I)+PW(I)/4.
            UPG(I)=UX
            IF (ILIFT.EQ.0) GO TO 140
            UX=UW(I)+UWT(I)-PW(I)/4.
            II=I+NWING
            UPG(II)=UX
140   CONTINUE
          RETURN
200  FORMAT (4X,31H CONFIGURATION ANGLE OF ATTACK=,F7.3,8H DEGREES)
      END

```

APPENDIX C - SAMPLE INPUT AND PRINTOUT

The subject computer program is dimensioned for a total of 100 panels to represent the wing planform and this limits the number of panels which can be used for the lifting case to 50. A symmetrical nonlifting wing, however, can be represented by a maximum of 100 panels. The storage requirement is approximately 145,000 in octal. The input data which defines the wing geometry and its relative position to the oncoming flow of given strength are arranged in the following sequence of namelists. These namelists are read in from the main program except the namelist LIFT which is read in from subroutine FORCE.

Note: The given numbers in the input data correspond to the sample calculation case shown in the printout. Namelist \$ SHOCK is only used for the output of the calculated locations of the shock and sonic lines in connection with METHOD = 3.

\$ WING

IWING = 0 Type of wing, i.e., arbitrary input (0), rectangular planform with parabolic arc cross-section (1), triangular planform with parabolic arc cross-section (2).

Note: An arbitrary half-wing planform is defined as a trapezoidal planform with the two parallel sides in the direction of the axis of symmetry (i.e., the x-axis). The coordinates of the four corner points is the required input, see ROOT(i) and TIP(i), together with the two thickness ratios THICK(1) and THICK(2).

INFLU = 1 Type of influence matrix to be used, i.e., semi two-directional (1), complete planar (2). INFLU = 0 for linearized solution only.

Note: It is recommended to use the option INFLU = 1 unless the program has been modified to account for a large number of wing panels, i.e., $N \approx 200-300$.

METHOD = 1 Method of calculation, i.e., Newton (1), parametric differentiation (2), method of steepest descent (3).

Note: METHOD = 1 should be preferred for purely subsonic flows whereas METHOD = 2 applies to slightly supercritical flows. For the calculation of discontinuous flow the option METHOD = 3 is the only one applicable.

ISECT = 3 Type of wing section, i.e., arbitrary input (0), parabolic arc (1), double wedge (2), NACA 00XX (3).

Note: The guidelines for arbitrary section input is described in Appendix B under subprogram name SECTIN.

LX = 10 Number of panel division in the chordwise direction.

LZ = 5	Number of panel division in the spanwise direction.	
ROOT (1) = 0.0	x-coordinate of the leading edge at the root.	} Not required for IWING = 1,2
ROOT (2) = 1.0	x-coordinate of the trailing edge at the root.	
ROOT (3) = 0.0	z-coordinate of the root section.	
ROOT (4) = 0.0	y-coordinate of the root section.	
TIP (1) = 0.0	x-coordinate of the leading edge at the tip.	
TIP (2) = 1.0	x-coordinate of the trailing edge at the tip.	
TIP (3) = 3.5	z-coordinate of the tip section.	
TIP (4) = 0.0	y-coordinate of the tip section.	
THICK (1) = 0.12	Thickness ratio of the root section.	
THICK (2) = 0.12	Thickness ratio of the tip section.	
SPAN = N.A.	Span to root-chord ratio for the cases of IWING = 1,2.	
TAU = N.A.	Thickness ratio for the cases of IWING = 1,2.	

\$ FLOW

MACH = 0.72	Initial freestream Mach number (M_∞).
DELM = 0.0	Desired increment of the freestream Mach number.
NDELM = 0	Number of increments desired of DELM.
Note: The options DELM and NDELM are introduced for the convenience of the user to reduce the number of input data cards for the case where only a change in Mach number is desired.	

\$ LIFT (required only for IWING = 0)

ILIFT = 0	Type of flow considered i.e., noncirculatory (0), circulatory (1).
CAMBER = 100 * 0.0	Local slope of each planar panel element which defines a cambered wing (in radians).
Note: The subscripted quantity CAMBER (i) is read in as an array of index $i = (m - 1)LX + n$ where $1 \leq n \leq LZ$ and $1 \leq m \leq LX$ and	

the indexing starts with the panel located at the leading edge of the root section. The input can be simplified by use of the notation shown for the sample case.

ALFA = 0.0

Angle of attack (in degrees).

The printout lists these namelists for each calculation case. The results from a flow calculation are written out in two rows for each chordwise calculation point. The upper row refers to the final nonlinear solution, whereas the lower row refers to the linearized or Prandtl-Glauert solution. The calculation points are located at the centroid of each wing panel and results are given for each spanwise station in the following notation:

I	Chordwise sequential number of wing panel
X/CHORD	Chordwise location of panel centroid with reference to local chordlength
U	Normalized perturbation velocity, $= \frac{u - u_{\infty}}{u_{\infty}}$
U + 1	Normalized velocity, $= \frac{u}{u_{\infty}}$
U (RED)	Reduced perturbation velocity, $= \frac{(\kappa + 1)M_{\infty}^2}{1 - M_{\infty}^2} \frac{u - u_{\infty}}{u_{\infty}}$
CP	Pressure coefficient, $= \frac{2}{\kappa M_{\infty}^2} \left\{ \left[1 + \frac{1}{2} (\kappa - 1) M_{\infty}^2 \left(1 - \frac{u^2}{u_{\infty}^2} \right) \right]^{\kappa/(\kappa-1)} - 1 \right\}$
CP (RED)	Reduced pressure coefficient, $= \frac{[(\kappa + 1)M_{\infty}^2]^{1/3}}{\tau^{2/3}} c_p$
P/PO	Pressure ratio, $= \frac{1}{2} \kappa M_{\infty}^2 c_p + 1$
M	Machnumber, $= M_{\infty} \frac{u}{u_{\infty}} \left\{ 1 + \frac{\kappa - 1}{2} M_{\infty}^2 \left[1 - \left(\frac{u}{u_{\infty}} \right)^2 \right] \right\}^{-1/2}$
M (STAR)	Critical Machnumber, $= M^*$

Note:
CP(STAR)

The critical value of the pressure coefficient (C_p^*) is evaluated from the exact relation.

4.

[illegible]

SEND.
CONFIGURATION ANGLE OF ATTACK = 0.000 DEGREES

PLANE DRH-
PROFILE-

THICKNESS RATIO = .12000
SEMI SPAN/CHORD = 3.50000
ASPECT RATIO = 7.00000

THE SOLUTION FOR THE WING

MACH NUMBER = .72000
BETA = .65397
MACH NUMBER (STAB) = .75076
CP (STAB) = -.69959
X1 = -1.71126

VELOCITY AND PRESSURE DISTRIBUTION ON UPPER SURFACE

1	X/CHORD	U	U+1	U(RED)	CP	CP(RED)	P/P0	M	M(STAR)
SPANWISE STATION, Z/CHORD = .35000 (Z/SEMI SPAN = .10000)									
1	.50000+00	-.26488+01	.97351+00	-.58429+01	.52529+01	.23266+00	.10191+01	.69934+00	.73387+03
		-.15617+01	.98438+00	-.40344+01	.31114+01	.13755+00	.10113+01	.70762+00	.73304+00
2	.15000+00	.21092+00	.12139+01	.50490+00	-.43983+00	-.13940+01	.84076+03	.89374+00	.93311+03
		.19121+00	.11912+01	.45358+00	-.39673+00	-.17559+01	.85603+00	.87693+00	.89432+00
3	.25000+00	.22070+00	.12237+01	.57015+00	-.45976+00	-.20325+01	.83316+03	.93212+00	.91345+03
		.19215+00	.11922+01	.45640+00	-.39737+00	-.17627+01	.85531+00	.87773+00	.89502+00
4	.35000+00	.19945+00	.11974+01	.49199+00	-.39509+00	-.17456+01	.85663+00	.87628+00	.89374+00
		.17181+00	.11718+01	.44386+00	-.35545+00	-.15714+01	.87101+00	.86152+00	.87975+00
5	.45000+00	.19427+00	.11543+01	.39853+00	-.31826+00	-.14070+01	.88451+03	.84577+00	.86558+00
		.14474+00	.11447+01	.37352+00	-.29814+00	-.13180+01	.89181+00	.83781+00	.85343+00
6	.55000+00	.11761+00	.11176+01	.30983+00	-.24111+00	-.10659+01	.91250+03	.81527+00	.83916+03
		.11481+00	.11148+01	.28655+00	-.23225+00	-.10400+01	.91463+00	.81296+00	.83655+00
7	.65000+00	.80854+01	.10879+01	.20888+00	-.16461+00	-.72771+00	.94027+03	.78539+00	.81146+00
		.62630+01	.10826+01	.21347+00	-.16826+00	-.74325+00	.93893+00	.78654+00	.81261+00
8	.75000+00	.41801+01	.10419+01	.10814+00	-.34332+01	-.37370+00	.96933+03	.75349+00	.78219+00
		.46282+01	.11463+01	.11957+00	-.33550+01	-.41357+00	.96605+00	.75755+00	.78351+00
9	.85000+00	-.55071+02	.39443+00	-.14382+01	.11119+01	.49156+01	.10043+01	.71558+00	.74553+00
		-.10650+02	.39853+00	-.27638+02	.21390+02	.94500+02	.11008+01	.71915+00	.74596+00
10	.95000+00	-.96479+01	.31352+00	-.22341+00	.16306+00	.74739+00	.10613+01	.85216+00	.88584+00
		-.25013+01	.30987+00	-.22485+00	.17602+00	.77815+00	.10639+01	.84933+00	.88309+00

SPANWISE STATION, Z/CHORD = 1.05000 (Z/SEMI SPAN = .30000)

1	.50000+00	-.26743+01	.97326+00	-.63086+01	.53132+01	.23489+00	.10193+01	.65884+00	.73568+00
		-.15595+01	.98406+00	-.41192+01	.31765+01	.14003+00	.10115+01	.73735+00	.73579+00
2	.15000+00	.21142+00	.12114+01	.50361+00	-.43776+00	-.17507+01	.84115+00	.89331+00	.90874+00
		.19097+00	.11909+01	.49310+00	-.39600+00	-.17507+01	.85630+03	.87664+00	.89436+00
3	.25000+00	.21999+00	.12216+01	.56832+00	-.45824+00	-.20288+01	.83371+00	.90151+00	.91592+00
		.19140+00	.11919+01	.49550+00	-.39799+00	-.17594+01	.85550+03	.87743+00	.89766+00
4	.35000+00	.19589+00	.11895+01	.49057+00	-.39391+00	-.17474+01	.85706+00	.87581+00	.89332+00
		.17147+00	.11715+01	.44297+00	-.38472+00	-.15681+01	.87128+03	.86322+00	.87343+00
5	.45000+00	.15584+00	.11538+01	.39742+00	-.31736+00	-.14050+01	.88484+00	.84541+00	.86626+00
		.14441+00	.11444+01	.37306+00	-.29744+00	-.13149+01	.89206+03	.83753+00	.85918+00
6	.55000+00	.11727+00	.11173+01	.31255+00	-.24041+00	-.10626+01	.91276+00	.81499+00	.83860+00
		.11449+00	.11145+01	.29578+00	-.23559+00	-.10371+01	.91487+03	.81273+00	.83572+00
7	.65000+00	.80578+01	.10816+01	.20817+00	-.16404+00	-.72519+00	.94047+00	.78487+00	.81126+00

SPANWISE STATION: Z/C-40RD = 1.75000 (Z/SEMI-SPAN = .50000)									
8	.82388-01	.10823+01	.21274+00	-.16770+00	-.74137+00	-.93915+00	.78631+00	.81258+00	
	.41636-01	.10416+01	.10156+00	-.83072-01	-.37117+00	.96909+00	.75333+00	.78232+00	
	.46037-01	.10416+01	.11893+00	-.93048-01	-.41135+00	.96623+00	.75668+00	.78532+00	
9	.85000+00	.39425+00	-.14846-01	.11777-01	.50718-01	.10042+01	.71544+00	.74445+00	
	-.57466-02	.39873+00	-.32910-02	.25470-02	.71260-01	.10004+01	.71890+00	.74980+00	
10	.95000+00	.91341+00	-.62370+00	.16928+00	.74834+00	.10614+01	.65238+00	.69575+00	
	-.96593-01	.91341+00	-.62370+00	.16928+00	.74834+00	.10614+01	.65238+00	.69575+00	
	-.50252-01	.91341+00	-.62370+00	.16928+00	.74834+00	.10614+01	.65238+00	.69575+00	

SPANWISE STATION: Z/C-40RD = 1.75000 (Z/SEMI-SPAN = .50000)

1	.50000-01	.97253+00	-.70969-01	.54570-01	.24124+00	.10198+01	.63826+00	.73014+00	
	-.18556-01	.98314+00	-.43548-01	.33573-01	.14892+00	.10122+01	.72664+00	.73911+00	
2	.15100+00	.12021+01	.25428+00	-.43501+00	-.19221+01	.84215+00	.89221+00	.90777+00	
	.18983+00	.11999+00	.43055+00	-.39390+00	-.17494+01	.85705+00	.87583+00	.89332+00	
3	.25100+00	.12185+01	.56454+00	-.45511+00	-.20120+01	.83485+00	.90026+00	.91482+00	
	.19077+00	.11978+00	.92836+00	-.39578+00	-.17497+01	.83578+00	.87655+00	.89398+00	
4	.35000+00	.11886+01	.88716+00	-.39110+00	-.17290+01	.85808+00	.87469+00	.89233+00	
	.17043+00	.11734+01	.49027+00	-.35250+00	-.15594+01	.87238+00	.85935+00	.87371+00	
5	.45100+00	.11527+01	.35439+00	-.24381+00	-.13920+01	.88578+00	.84443+00	.86538+00	
	.14340+00	.11430+01	.37045+00	-.24531+00	-.13055+01	.89289+00	.83529+00	.85842+00	
6	.55000+00	.11163+01	.31033+00	-.23828+00	-.10534+01	.91353+00	.81415+00	.83604+00	
	.11355+00	.11135+01	.29335+00	-.23263+00	-.10294+01	.91550+00	.81192+00	.83331+00	
7	.65000+00	.10757+01	.20599+00	-.16230+00	-.71750+00	.94100+00	.78418+00	.81162+00	
	.79736-01	.10757+01	.20599+00	-.16230+00	-.71750+00	.94100+00	.78418+00	.81162+00	
8	.75000+00	.10515+01	.14059+00	-.16398+00	-.73376+00	.93977+00	.78556+00	.81156+00	
	.40592-01	.10416+01	.12087+00	-.82741-01	-.36578+00	.96958+00	.75278+00	.79153+00	
9	.85000+00	.10453+01	.11713+00	-.91629-01	-.40508+00	.96675+00	.75529+00	.79993+00	
	.562198-02	.99576+00	-.16168-01	.12821-01	.54930-01	.10045+01	.71506+00	.74603+00	
10	.95000+00	.99513+00	-.45858-02	.35259-02	.16030-01	.10013+01	.71856+00	.74943+00	
	-.66674-01	.91313+00	-.22443+00	.16981+00	.75671+00	.10615+01	.65186+00	.68554+00	
	-.90689-01	.90933+00	-.23423+00	.17704+00	.78265+00	.10642+01	.64892+00	.69269+00	

SPANWISE STATION: Z/C-40RD = 2.45000 (Z/SEMI-SPAN = .70000)

1	.50000-01	.97065+00	-.75787-01	.59246-01	.25750+00	.13211+01	.59679+00	.72874+00	
	-.19336-01	.98074+00	-.49758-01	.38339-01	.16989+00	.10139+01	.70474+00	.73630+00	
2	.15000+00	.12055+01	.53077+00	-.42713+00	-.18893+01	.84500+00	.88935+00	.91331+00	
	.18636+00	.11870+01	.48300+00	-.38767+00	-.17138+01	.85932+00	.87353+00	.89113+00	
3	.25000+00	.12139+01	.55263+00	-.44524+00	-.19683+01	.83843+00	.89633+00	.91156+00	
	.21392+00	.11873+01	.48431+00	-.38875+00	-.17156+01	.85893+00	.87375+00	.89151+00	
4	.35000+00	.11842+01	.47576+00	-.38170+00	-.16874+01	.86149+00	.87395+00	.89232+00	
	.16636+00	.11670+01	.43131+00	-.34514+00	-.15258+01	.87476+00	.85642+00	.87611+00	
5	.45000+00	.11487+01	.38914+00	-.30649+00	-.13550+01	.88878+00	.84111+00	.85243+00	
	.14970+00	.11410+01	.36169+00	-.28816+00	-.12735+01	.89543+00	.83866+00	.85587+00	
6	.55000+00	.11123+01	.23173+00	-.23131+00	-.10226+01	.91635+00	.81143+00	.83554+00	
	.11048+00	.11072+01	.28541+00	-.22620+00	-.99999+00	.91792+00	.80539+00	.83370+00	
7	.65000+00	.10772+01	.19937+00	-.15700+00	-.69409+00	.94373+00	.78239+00	.83970+00	
	.78555-01	.10750+01	.11397+00	-.10668+00	-.71035+00	.94169+00	.78354+00	.81004+00	
8	.75000+00	.10392+01	.10137+00	-.79188-01	-.35008+00	.97126+00	.75137+00	.78222+00	
	.39233-01	.10392+01	.10137+00	-.79188-01	-.35008+00	.97126+00	.75137+00	.78222+00	
9	.85000+00	.10434+01	.11220+00	-.87733-01	-.38785+00	.96816+00	.75475+00	.78337+00	
	.72087-02	.99273+00	-.18623-01	.14392-01	.63625-01	.10352+01	.71429+00	.74335+00	
10	.95000+00	.99650+00	-.22521-02	.60275-02	.26687-01	.10022+01	.71760+00	.74850+00	
	-.97175-01	.91283+00	-.22521+00	.17038+00	.75324+00	.10618+01	.65163+00	.69331+00	
	-.95184-01	.91862+00	-.23556+00	.17801+00	.78657+00	.10646+01	.64852+00	.69230+00	

SPANWISE STATION: Z/C-40RD = 3.15000 (Z/SEMI-SPAN = .90000)

1	.50000-01	.96671+00	-.65127-01	.65364-01	.28886+00	.10237+01	.69395+00	.72602+00	
	-.232952-01	.97479+00	-.65139-01	.50115-01	.22155+00	.10182+01	.69304+00	.73133+00	

2	.15000+00	.14990+00	.11899+01	.49034+00	-.39372+00	-.17406+01	.89713+00	.87574+00	.89326+00
3	.25000+00	.17566+00	.11737+01	.44864+00	-.35938+00	-.15888+01	.86959+00	.86208+00	.88114+00
4	.35000+00	.18914+00	.11391+01	.48804+00	-.39018+00	-.17249+01	.85841+00	.87433+00	.89231+00
5	.45000+00	.16916+00	.11652+01	.43711+00	-.34982+00	-.15465+01	.87306+00	.85828+00	.87776+00
6	.55000+00	.15832+00	.11583+01	.40901+00	-.32684+00	-.14442+01	.86147+00	.84917+00	.85962+00
7	.65000+00	.14700+00	.11470+01	.37375+00	-.30290+00	-.13393+01	.85006+00	.83569+00	.86112+00
8	.75000+00	.12651+00	.11265+01	.32881+00	-.25977+00	-.11489+01	.80574+00	.82654+00	.84574+00
9	.85000+00	.12117+00	.11212+01	.31913+00	-.24857+00	-.10989+01	.80980+00	.81822+00	.84173+00
10	.95000+00	.98002+01	.10950+01	.24801+00	-.19603+00	-.86661+00	.92887+00	.79749+00	.82284+00
11	.65000+00	.67338+01	.10947+01	.24075+00	-.19396+00	-.85499+00	.92982+00	.79645+00	.82189+00
12	.75000+00	.67662+01	.10662+01	.17108+00	-.13442+00	-.59425+00	.95122+00	.77313+00	.83148+00
13	.85000+00	.67662+01	.10677+01	.17480+00	-.13738+00	-.60735+00	.95015+00	.77435+00	.80156+00
14	.95000+00	.57451+01	.10375+01	.80404+01	-.69300+01	-.30636+00	.97485+00	.74745+00	.77557+00
15	.85000+00	.57451+01	.10375+01	.86751+01	-.75594+01	-.33401+00	.97258+00	.74994+00	.77888+00
16	.75000+00	.55623+02	.99344+00	.16353+01	.13104+01	.57929+01	.10048+01	.71479+00	.74593+00
17	.65000+00	.55623+02	.99344+00	.16353+01	.13104+01	.57929+01	.10048+01	.71479+00	.74593+00
18	.55000+00	.53966+02	.99660+00	.87746+02	.67876+02	.30007+01	.10025+01	.71750+00	.74821+00
19	.45000+00	.52672+01	.91733+00	.21357+00	.16179+00	.71526+00	.10587+01	.55512+00	.68869+00
20	.35000+00	.50695+01	.91330+00	.22357+00	.16947+00	.74920+00	.10615+01	.65200+00	.68367+00

THE LINEARIZED SOLUTION IS BASED ON
CARMICHAEL'S NUMERICAL ANALYSIS

THE NONLINEAR SOLUTION WAS OBTAINED WITH
THE NEWTON'S METHOD
AFTER 3 ITERATIONS

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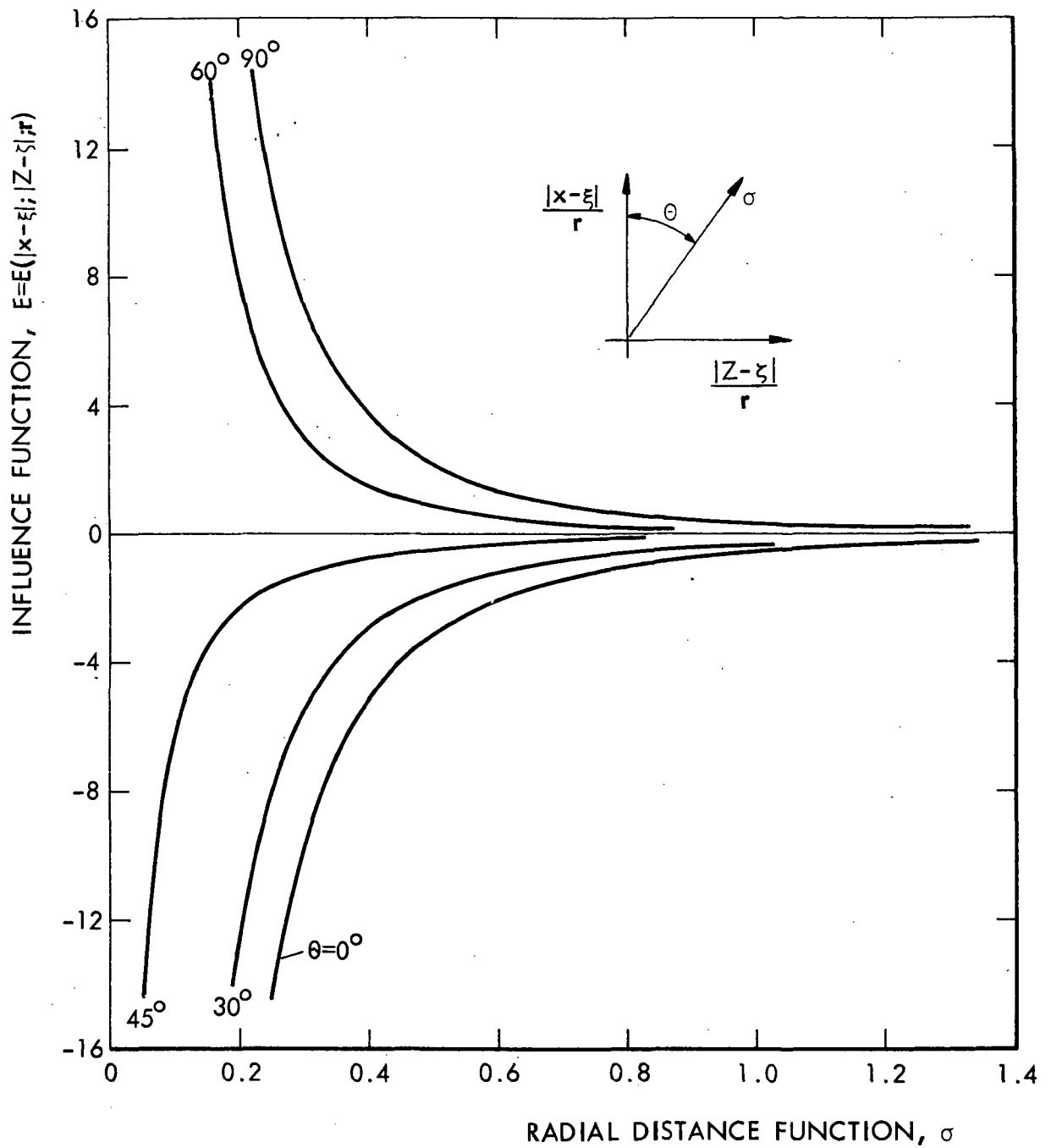


FIGURE 1 - Influence function E for various angular deviation from the free-stream direction.

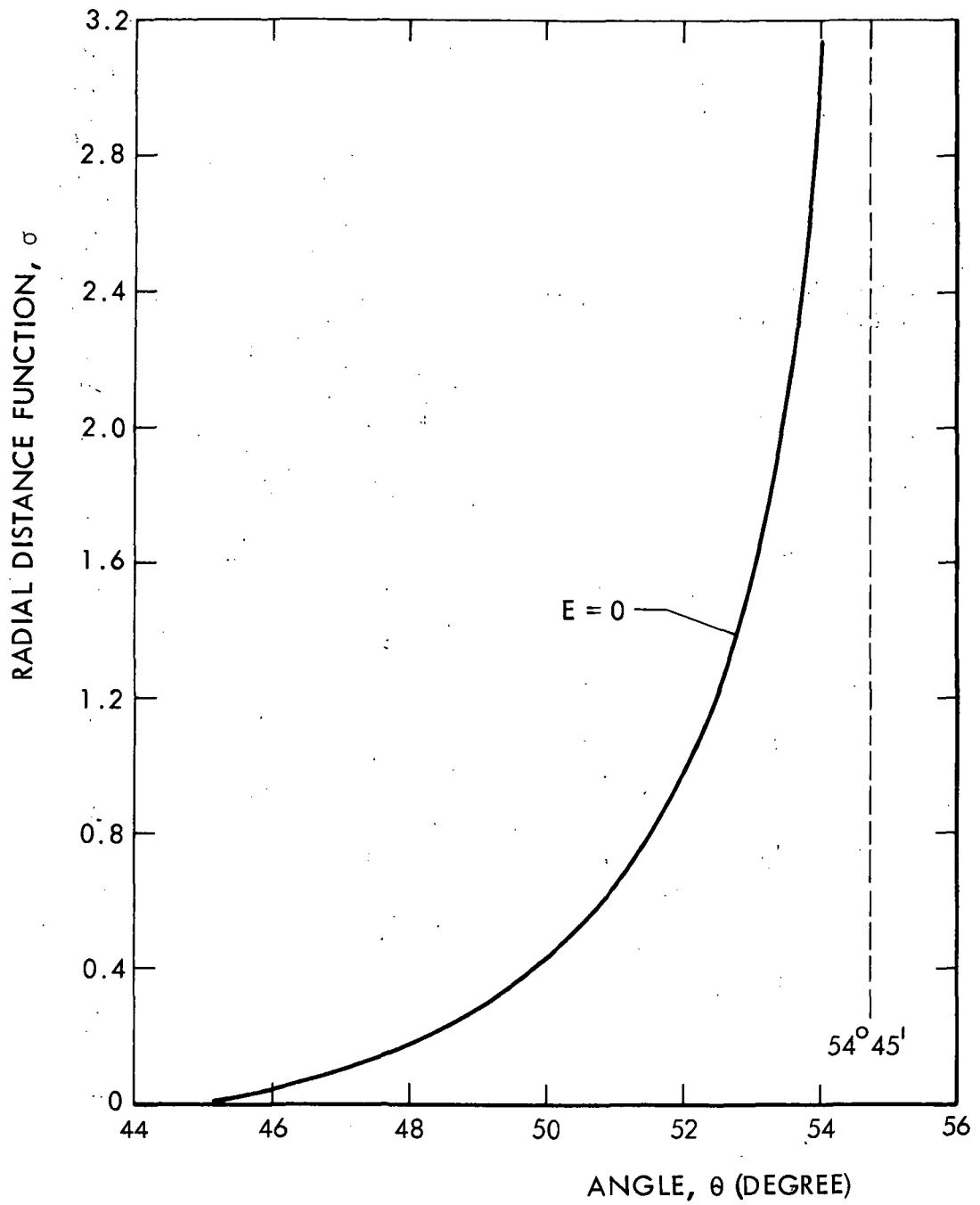


FIGURE 2 - Location of zero influence line.

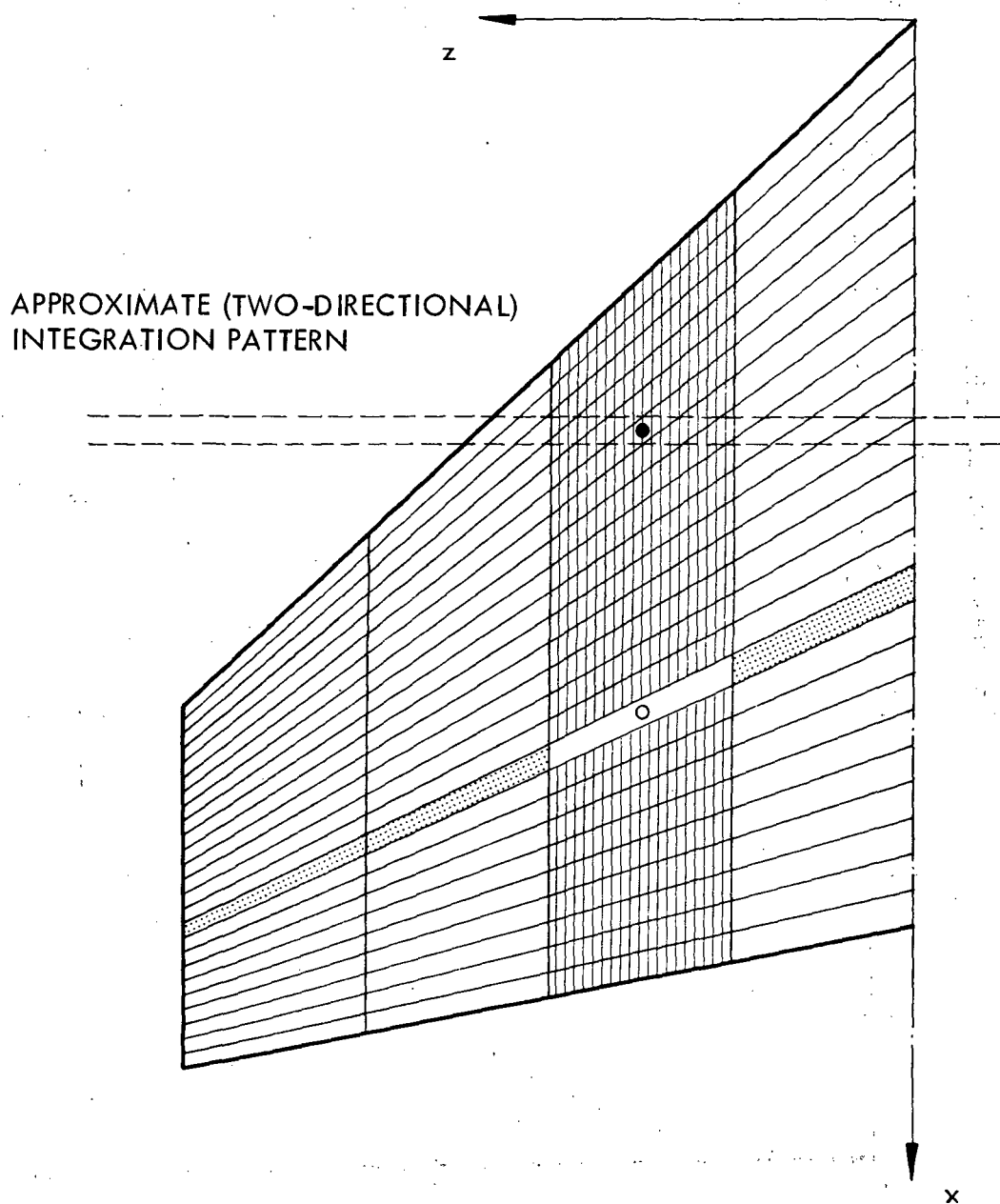


FIGURE 3 - Geometric division of an arbitrary planform into a number of trapezoidal panels.

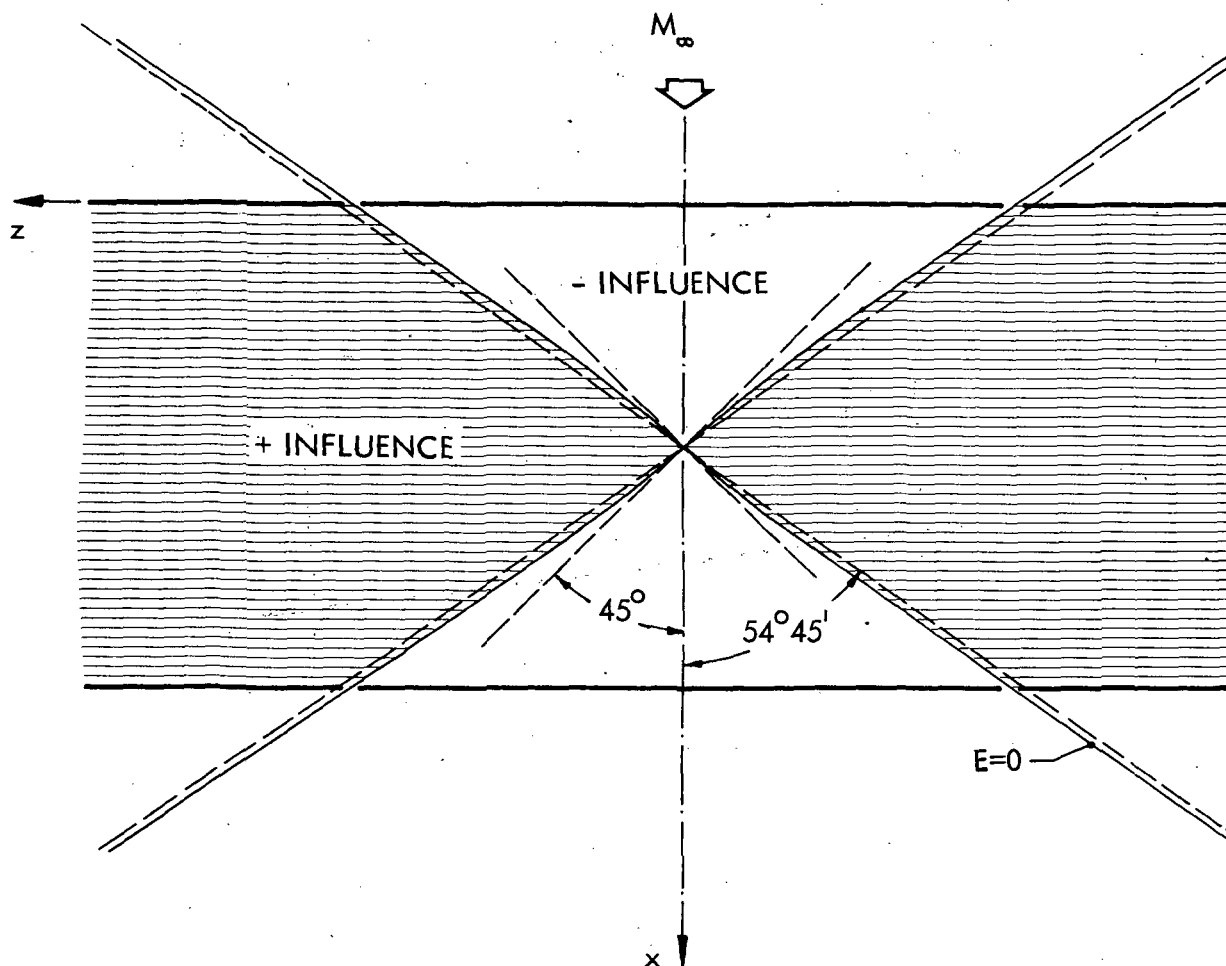


FIGURE 4 - Influence pattern with reference to the mid-point of a rectangular wing.

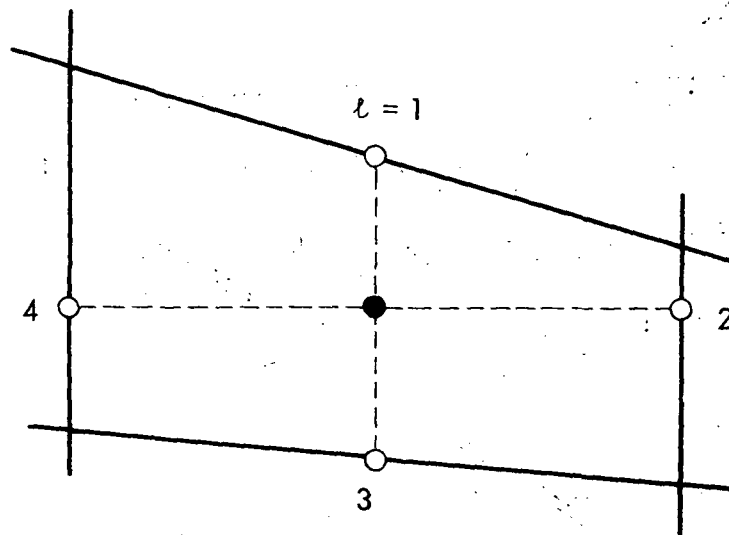


FIGURE 5 - Integration points for the definition of the influence measure from an elementary panel.

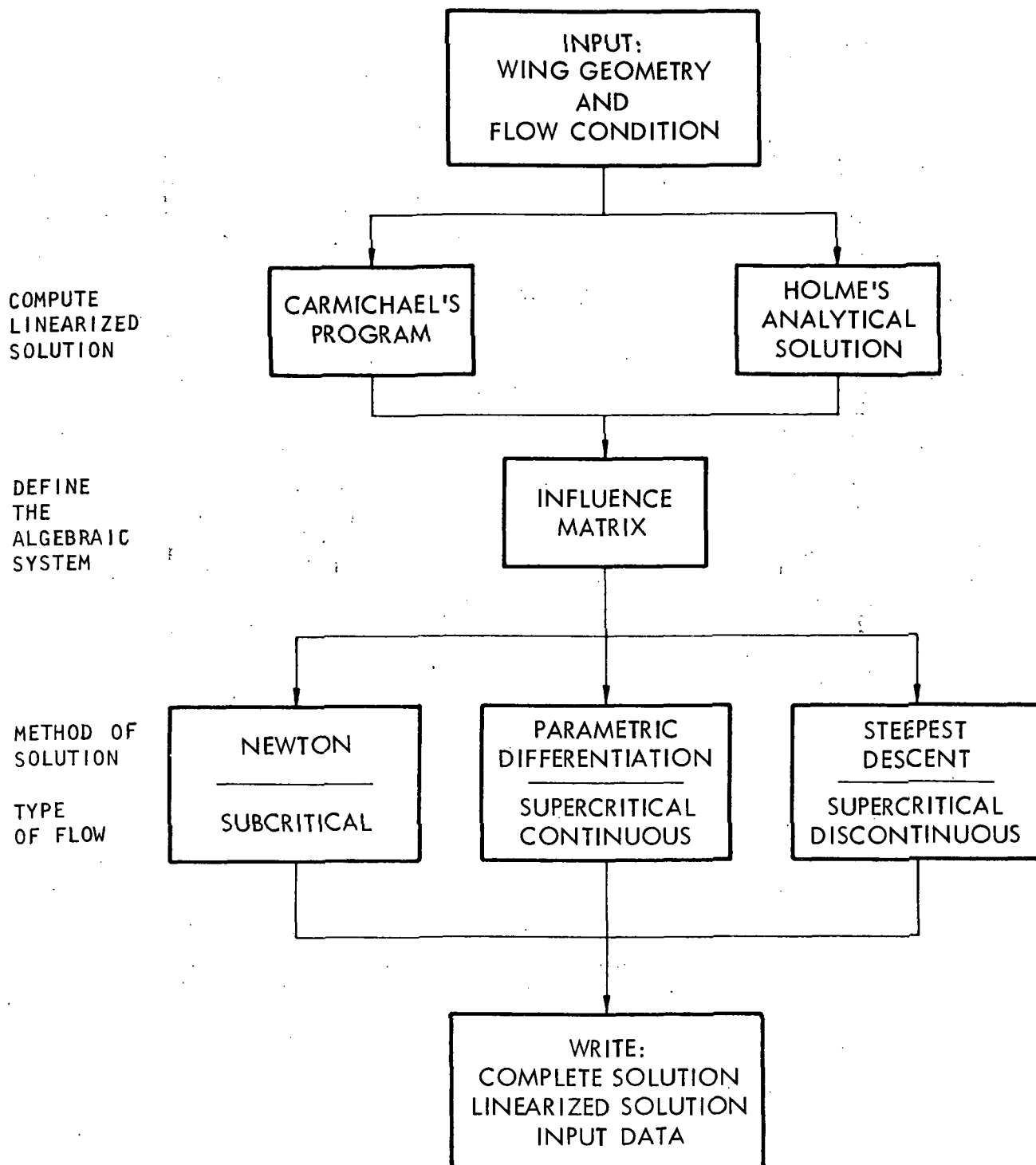


FIGURE 6 - Block diagram of the computer program.

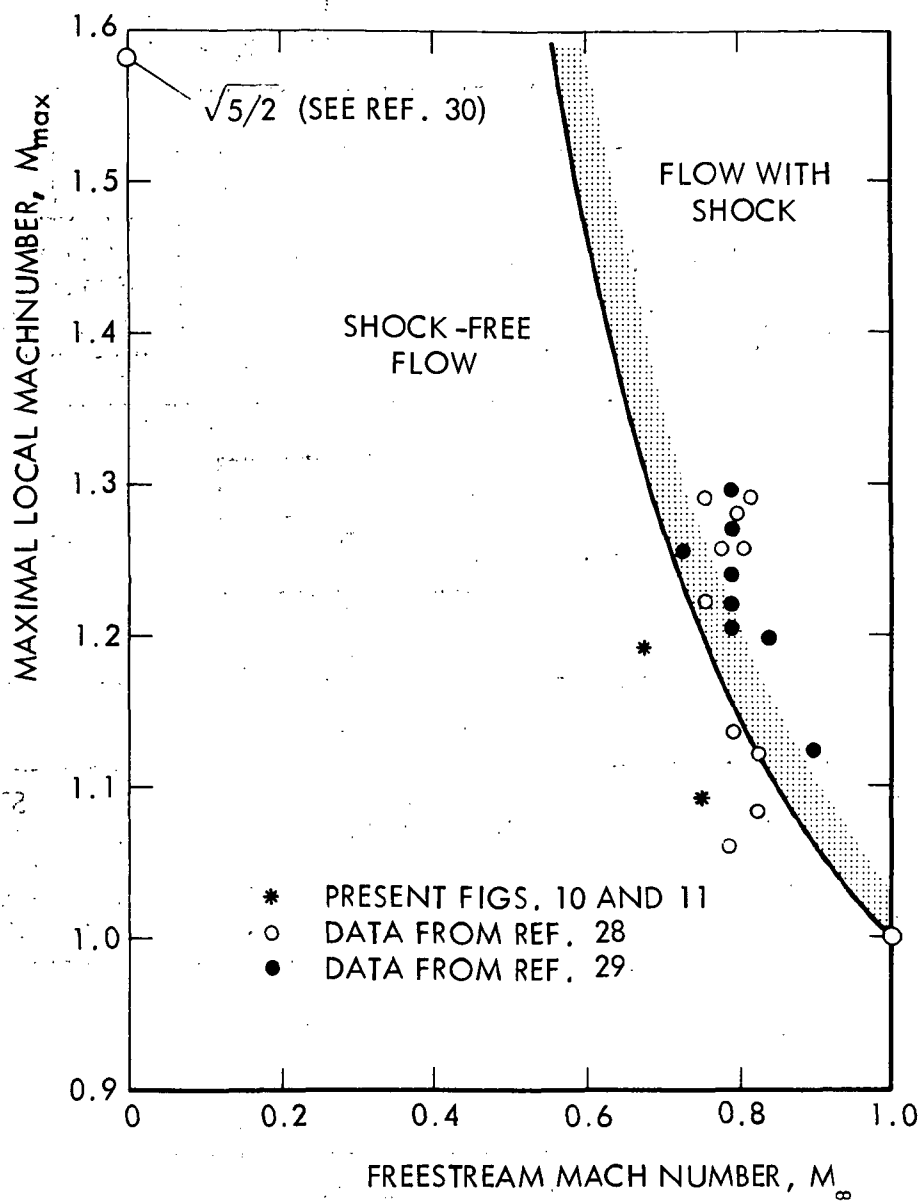


FIGURE 7 - Limits for shock-free transonic flow.

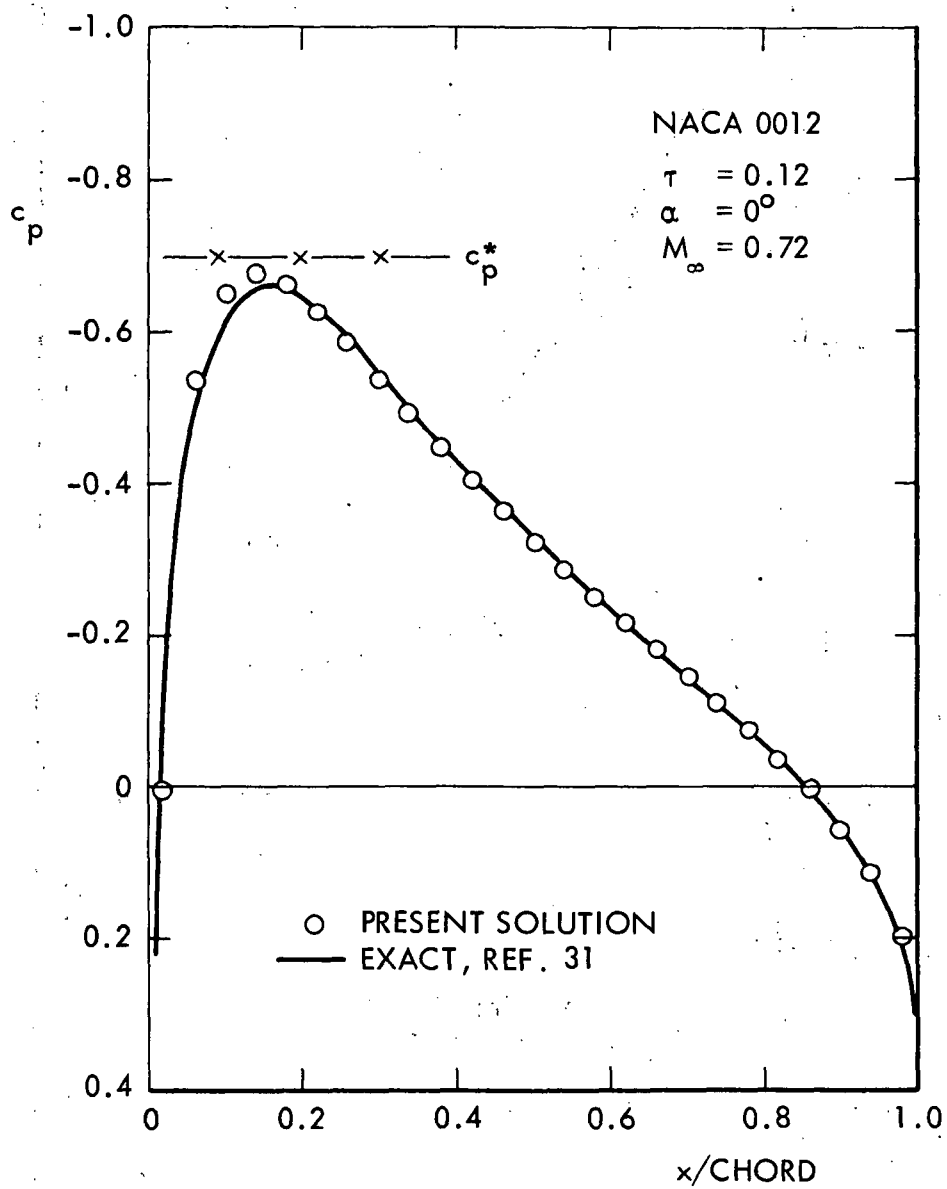


FIGURE 8 - Subcritical flow past a non-lifting airfoil.

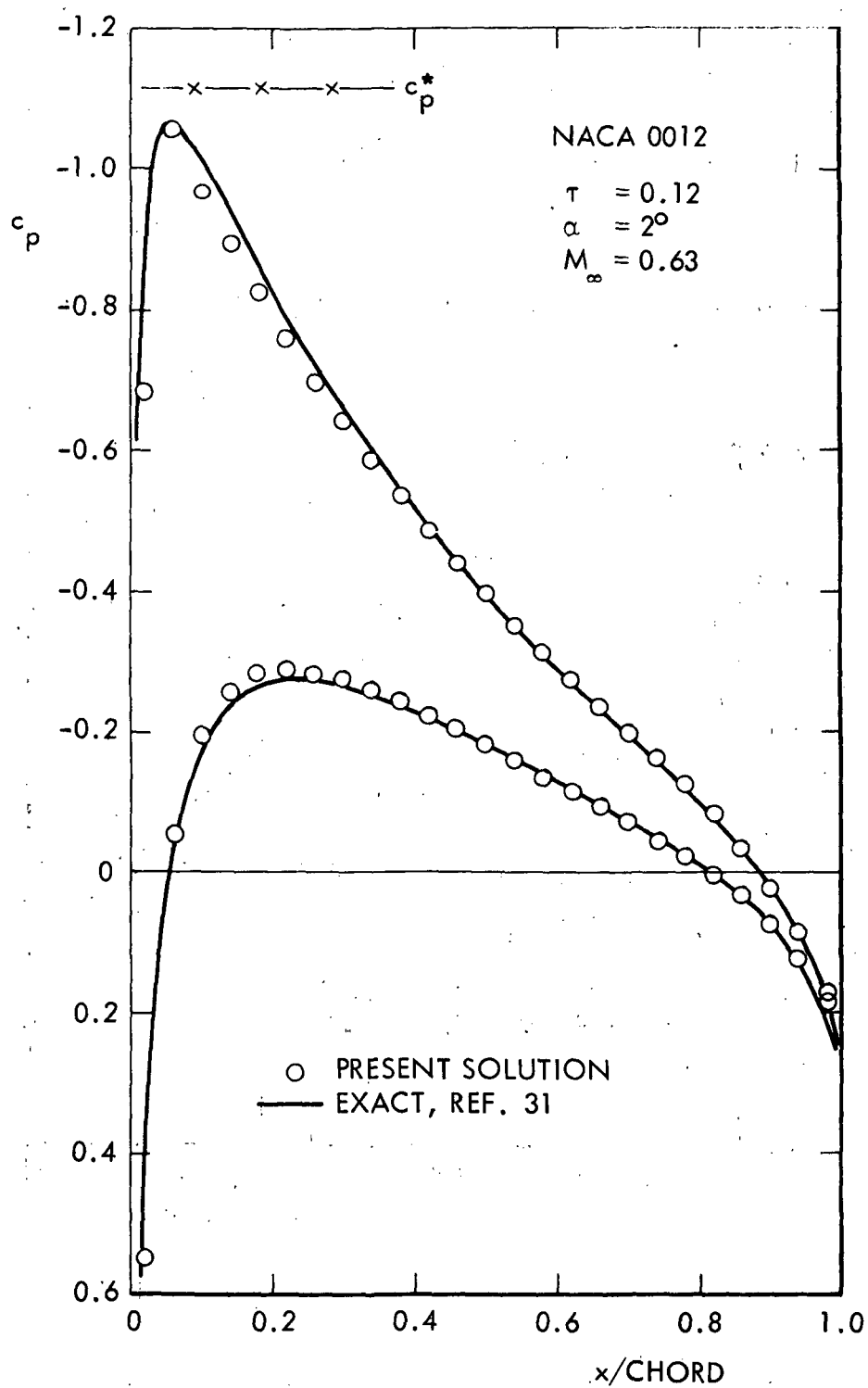


FIGURE 9 - Subcritical flow past a lifting airfoil.

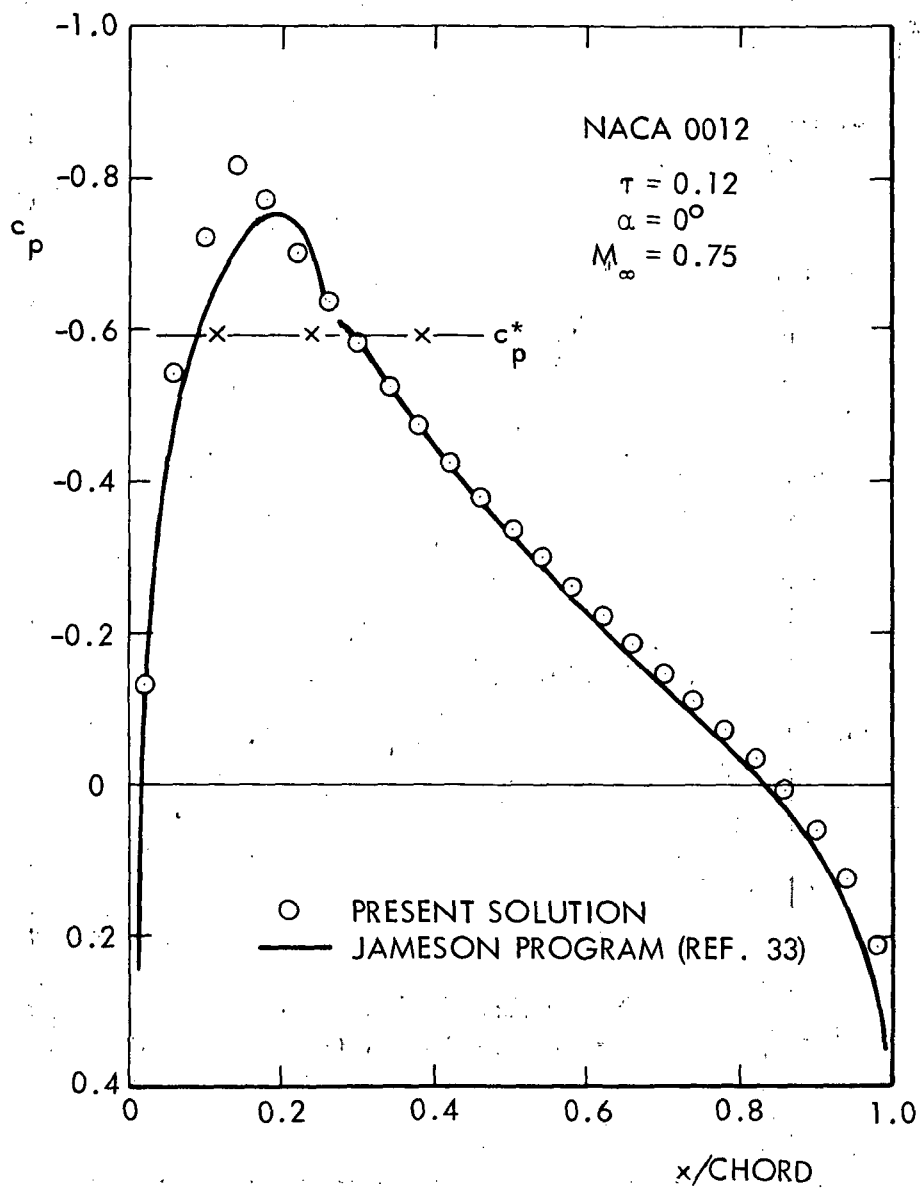


FIGURE 10 - Supercritical flow past a non-lifting airfoil.

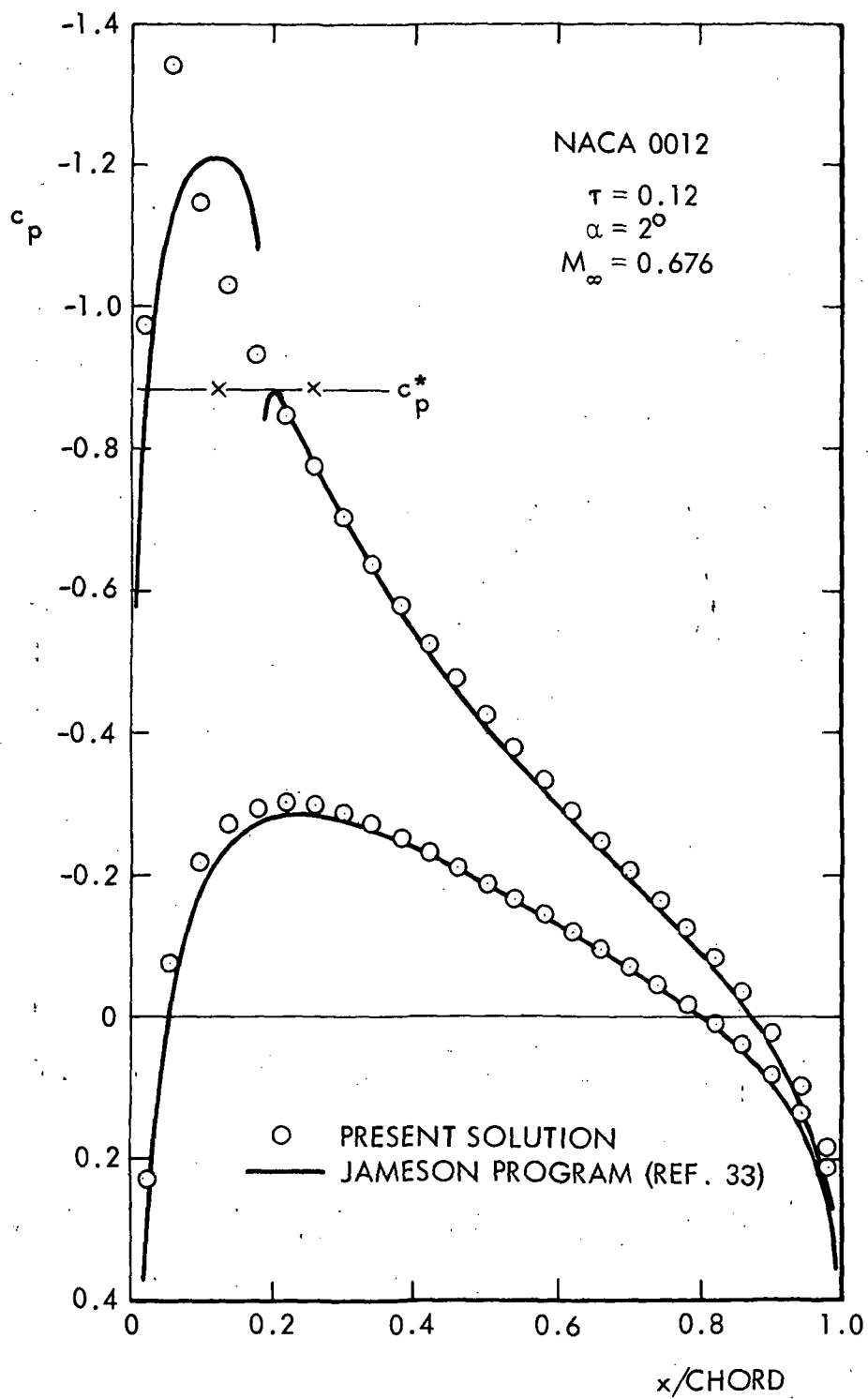


FIGURE 11 - Supercritical flow past a lifting airfoil.

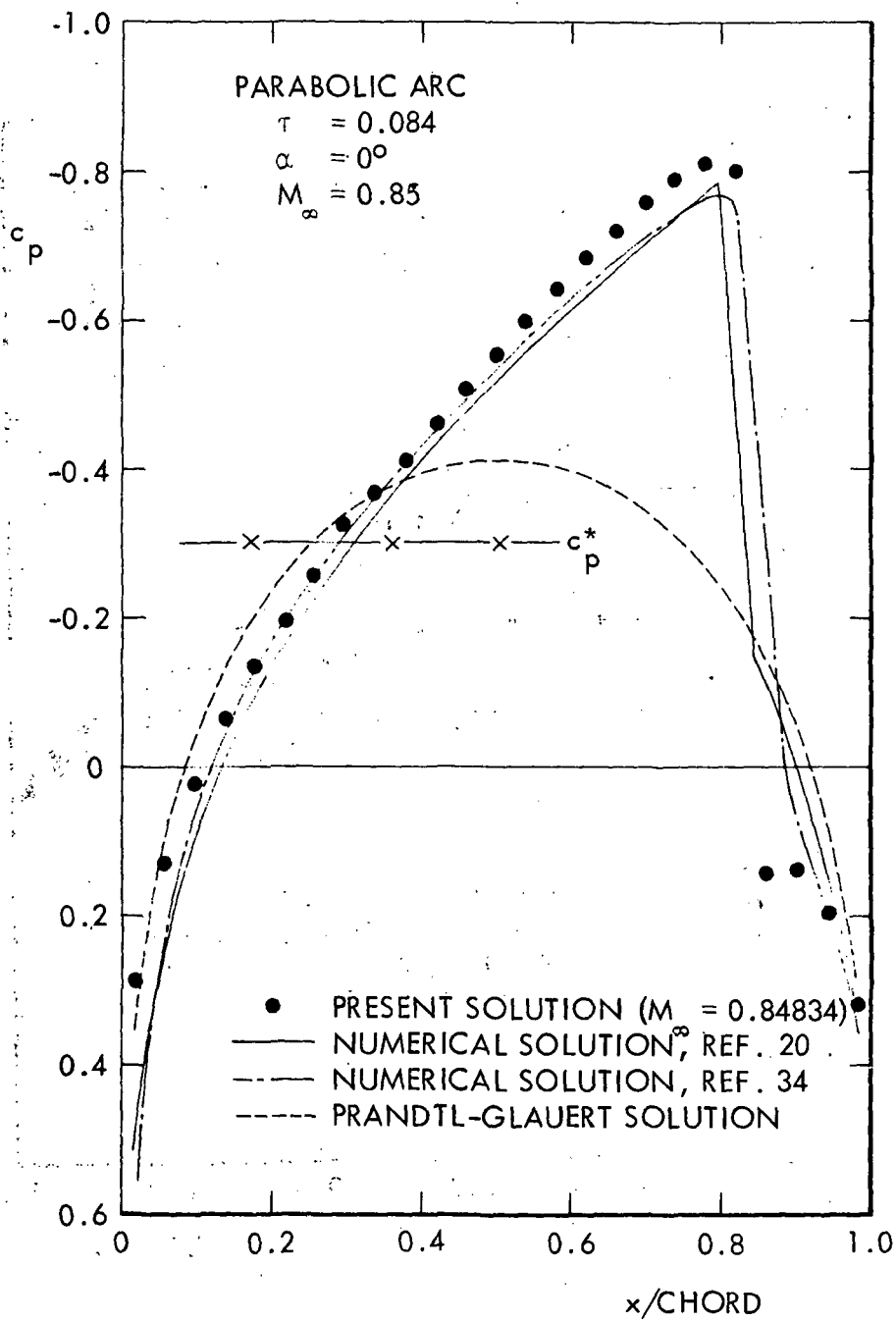


FIGURE 12 - Discontinuous flow past a non-lifting airfoil.

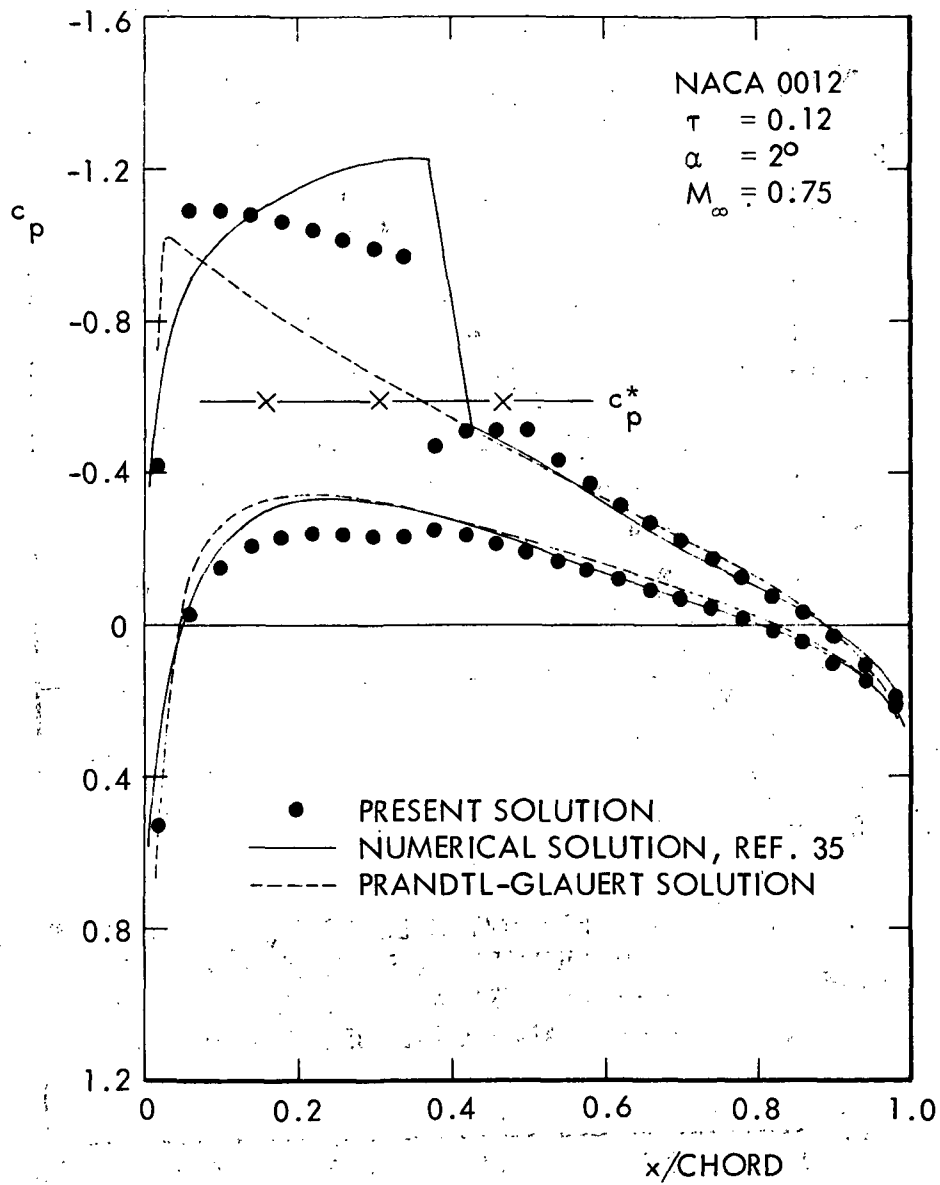


FIGURE 13 - Discontinuous flow past a lifting airfoil.

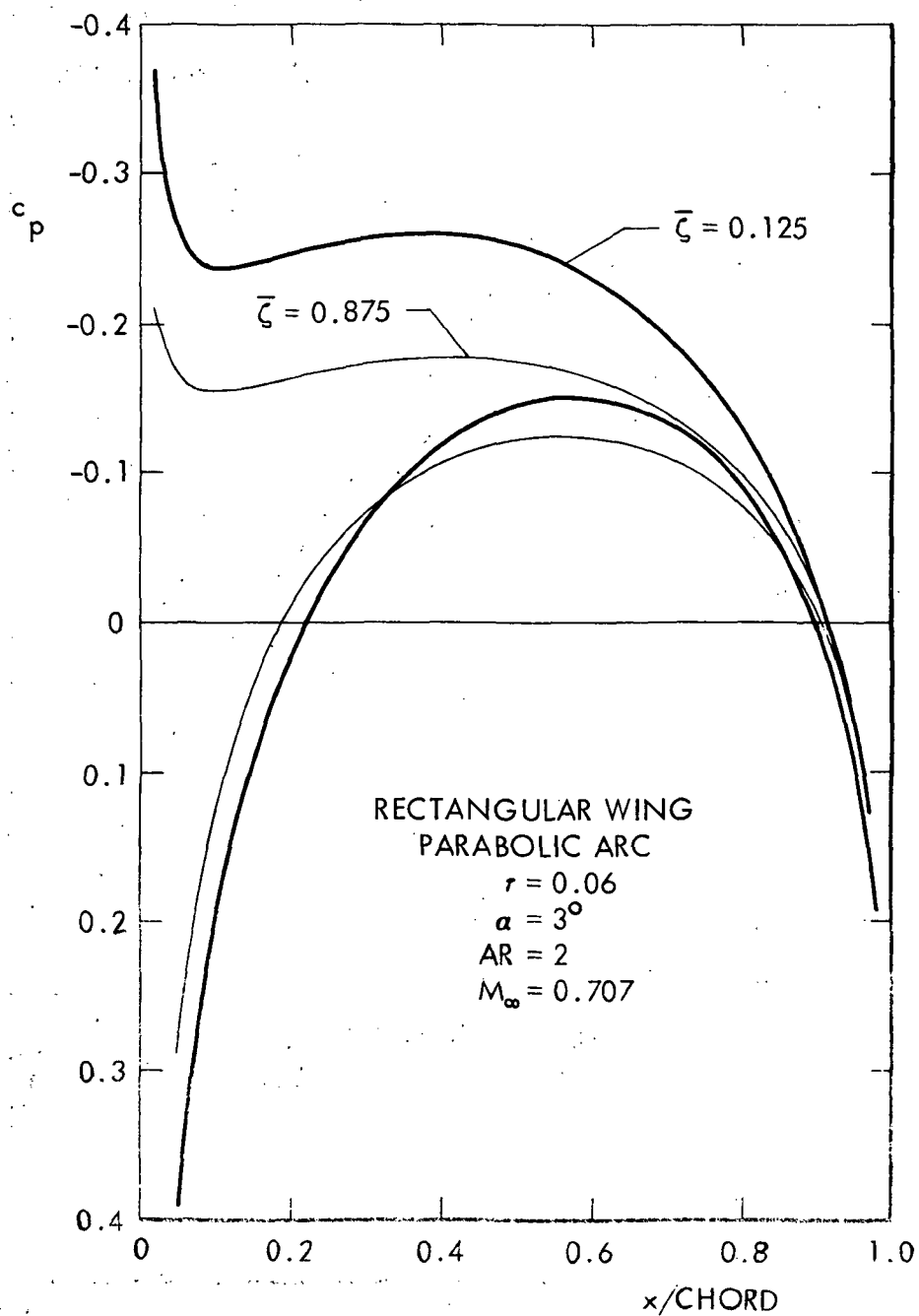


FIGURE 14 - Linearized solution for a lifting wing.

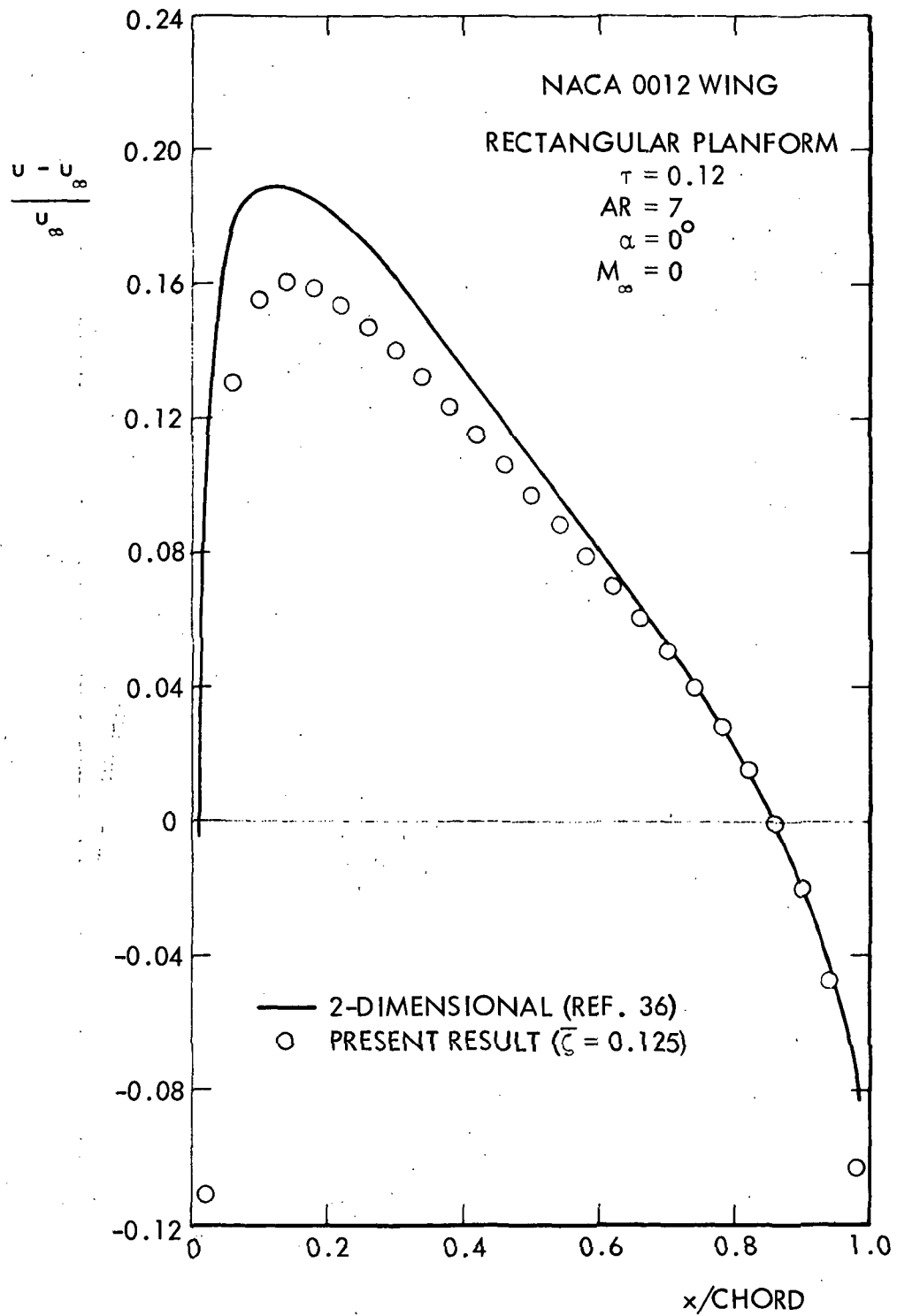


FIGURE 15 - Comparisons of analytical and numerical results for a symmetrical wing.

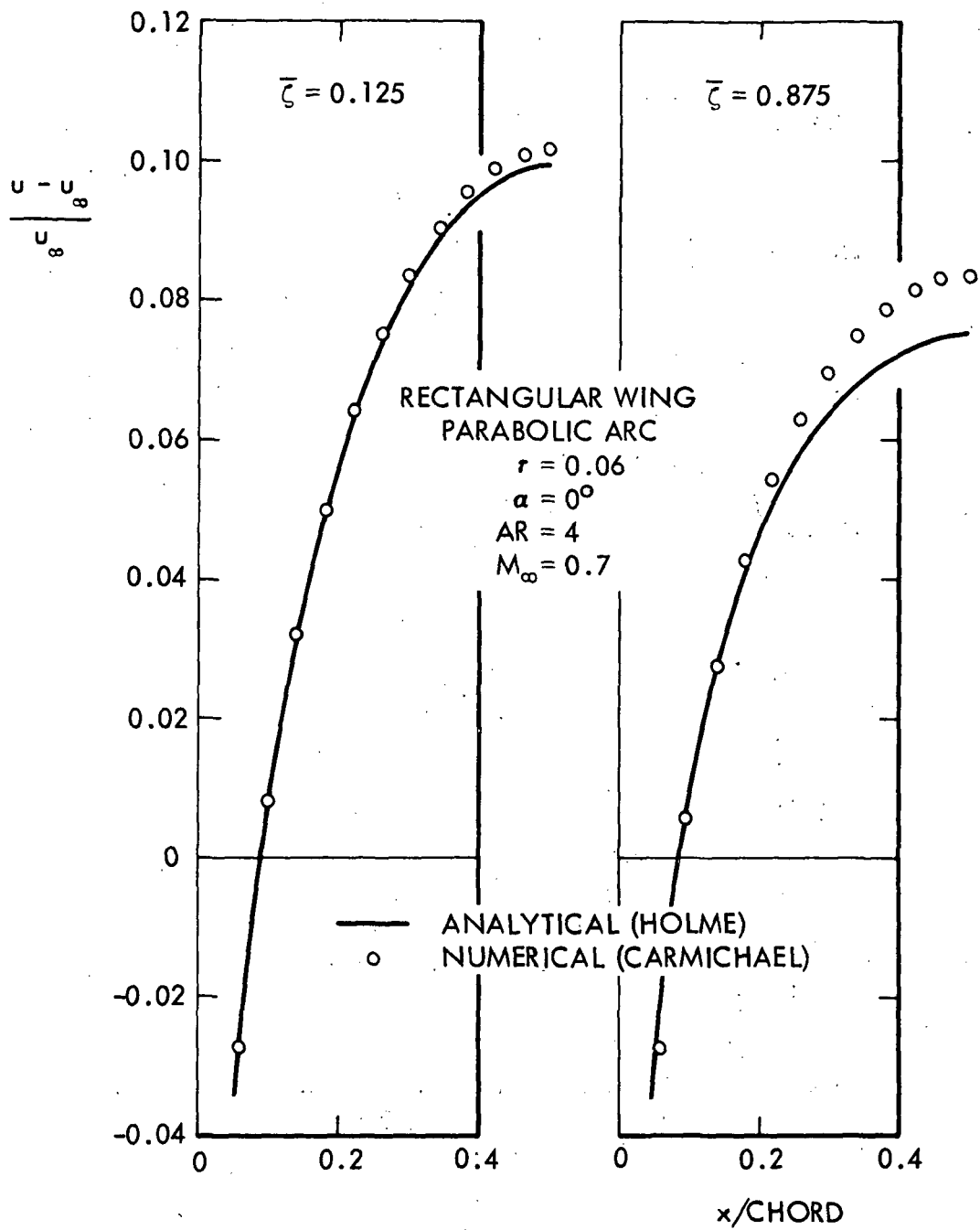


FIGURE 16 - Incompressible flow solutions for high aspect ratio wings with a blunt leading edge.

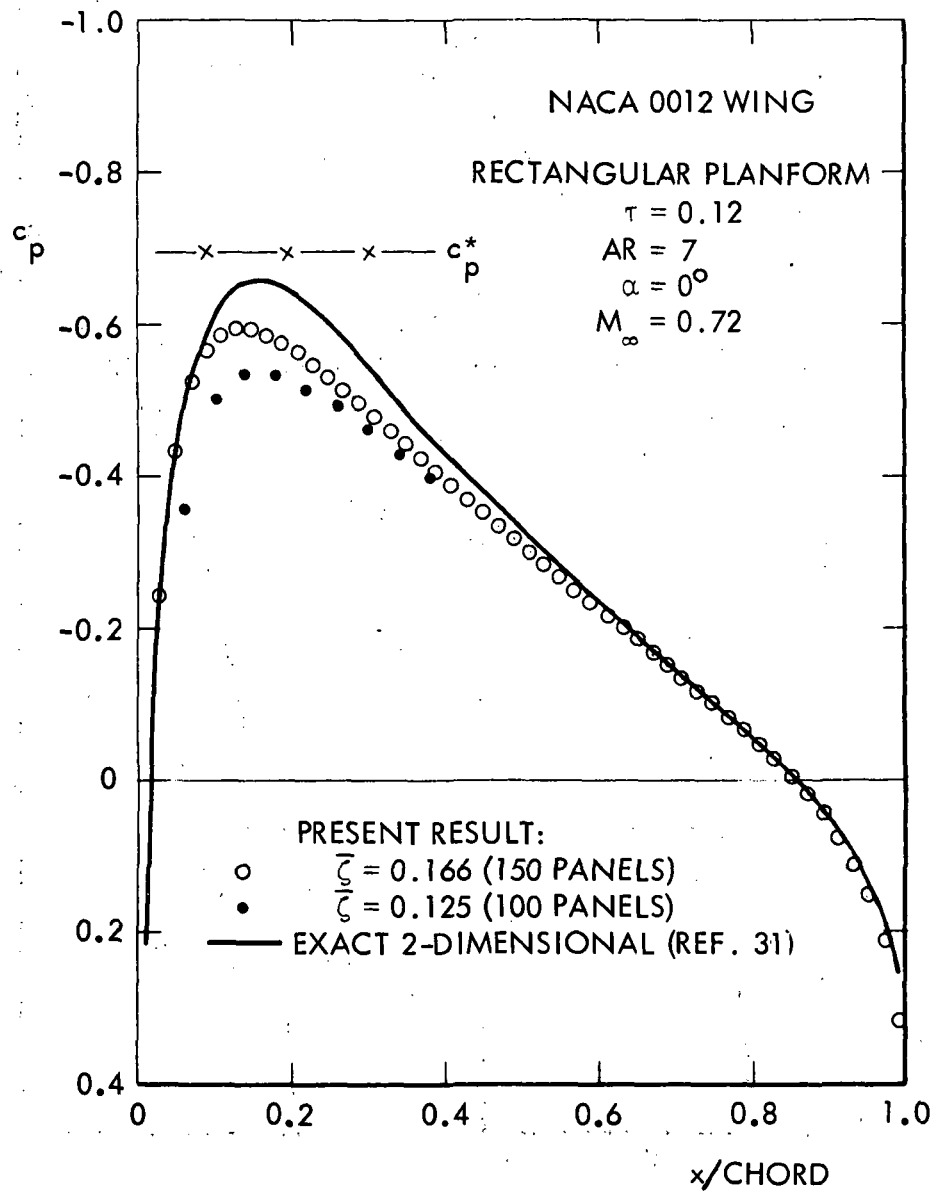


FIGURE 17 - Subcritical flow past a non-lifting rectangular high aspect ratio wing.

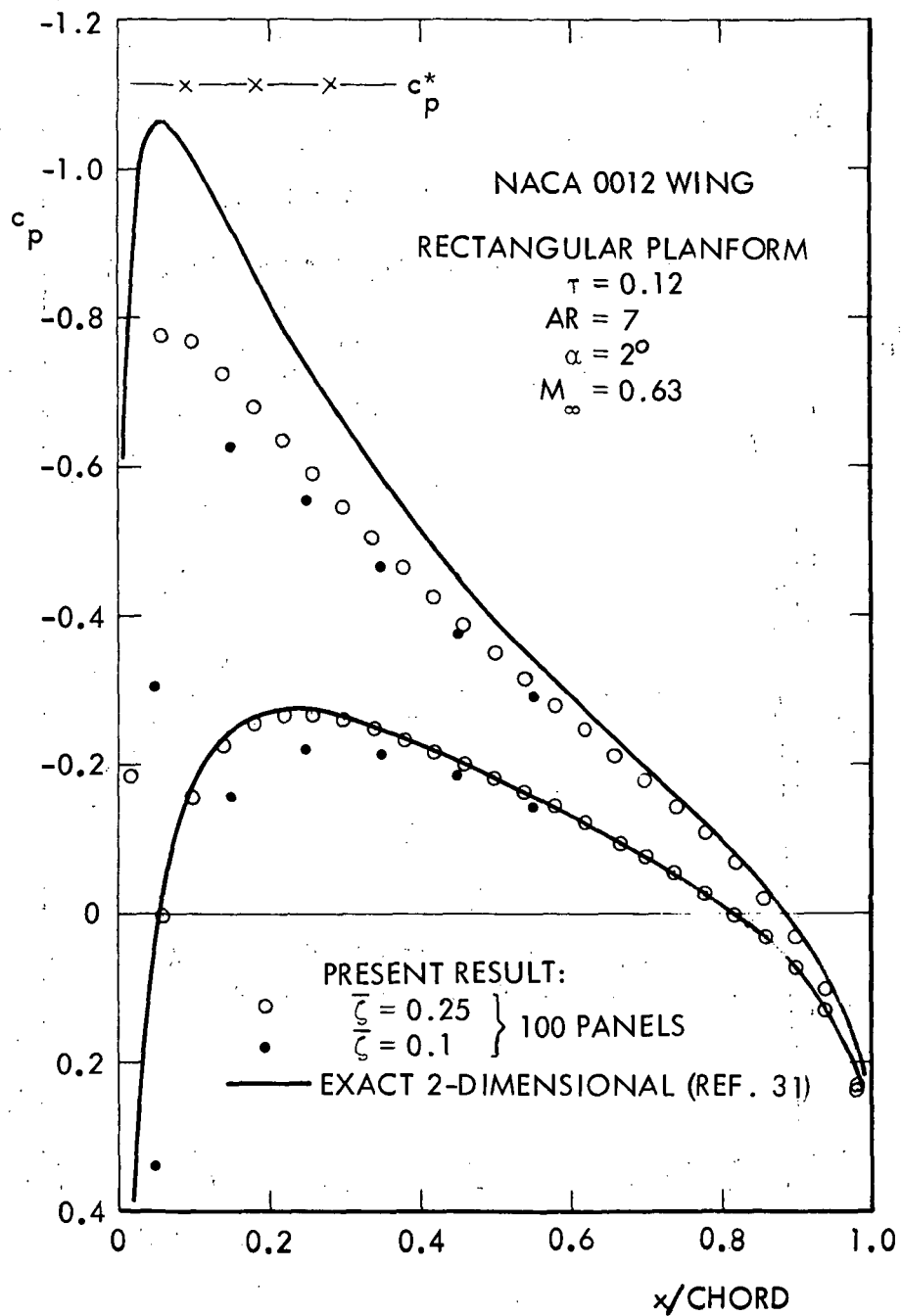


FIGURE 18 - Subcritical flow past a lifting rectangular high aspect ratio wing.

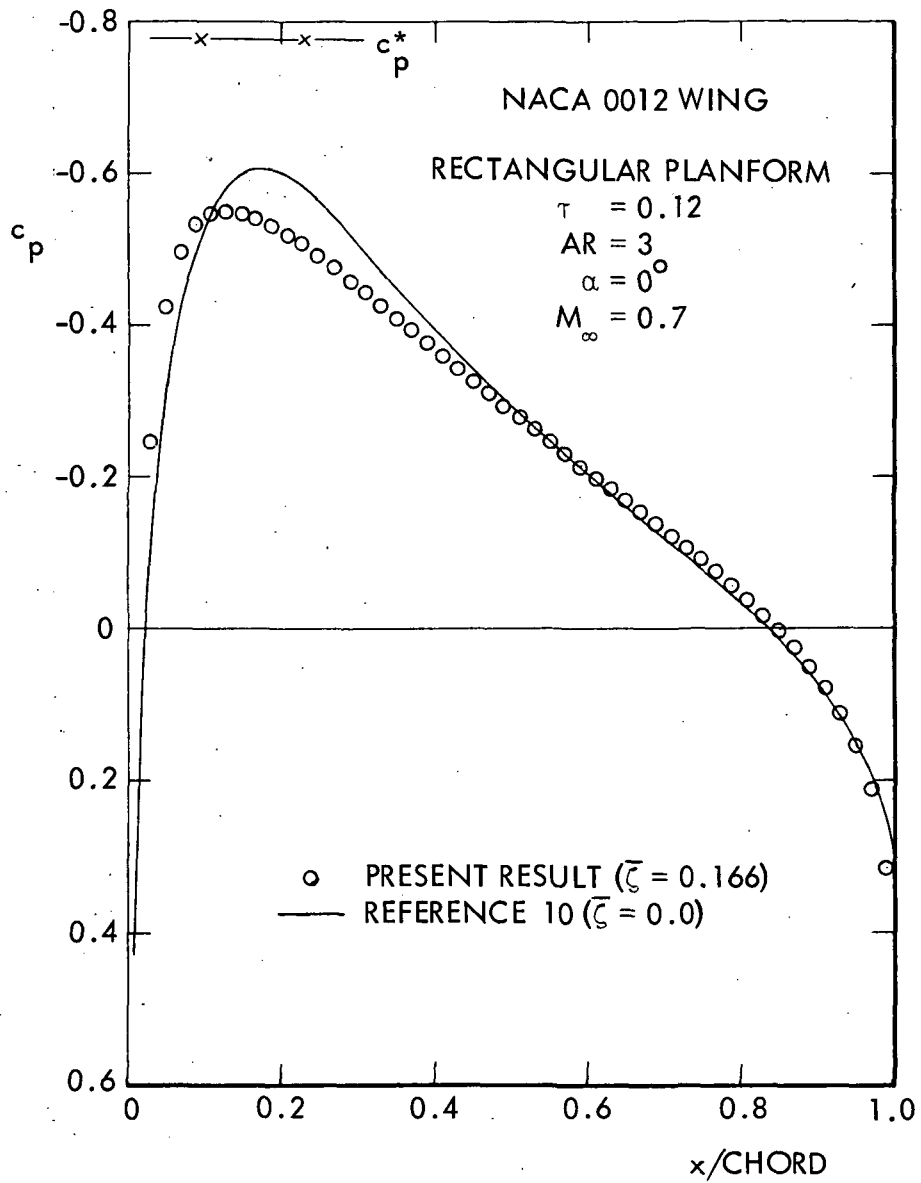


FIGURE 19 - Subcritical flow past a non-lifting rectangular wing.

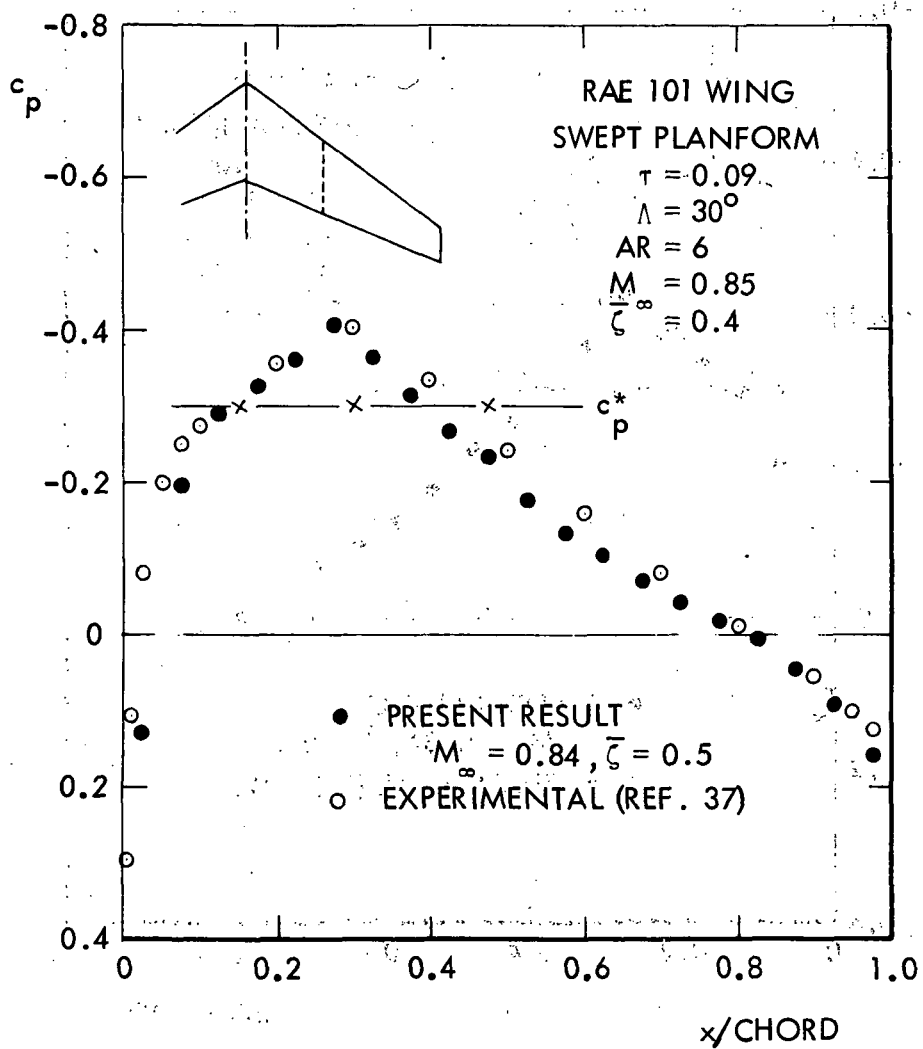


FIGURE 20 - Continuous supercritical flow past a non-lifting swept wing ($\bar{\zeta} = 0.4$).

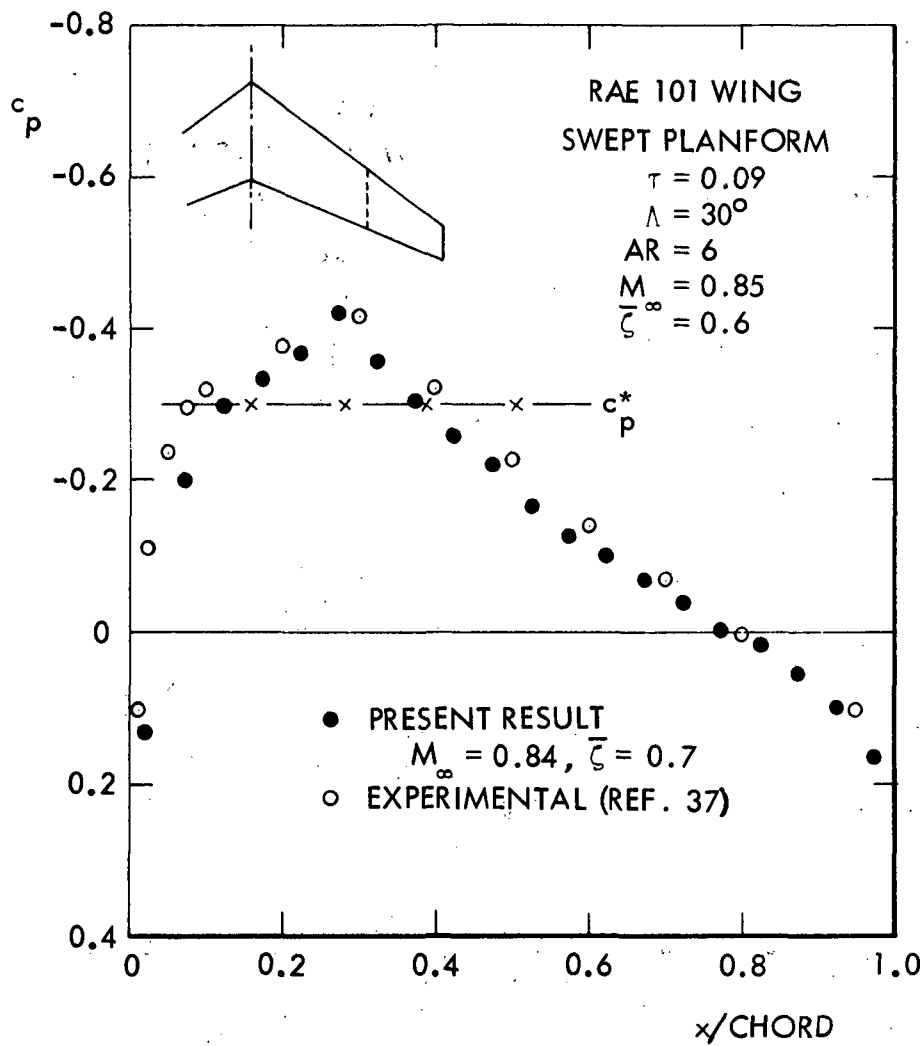


FIGURE 21 - Continuous supercritical flow past a non-lifting swept wing ($\bar{\zeta} = 0.6$).

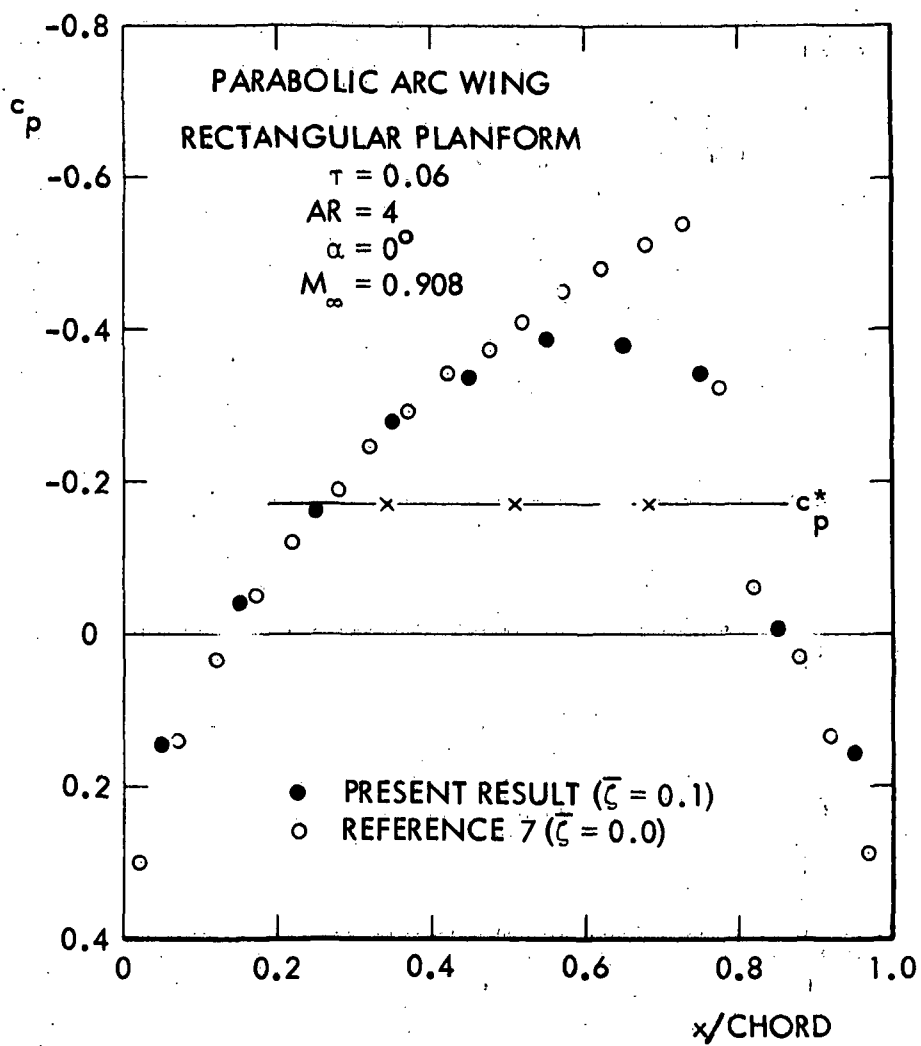


FIGURE 22 - Discontinuous flow past a non-lifting rectangular wing near the mid-span station.

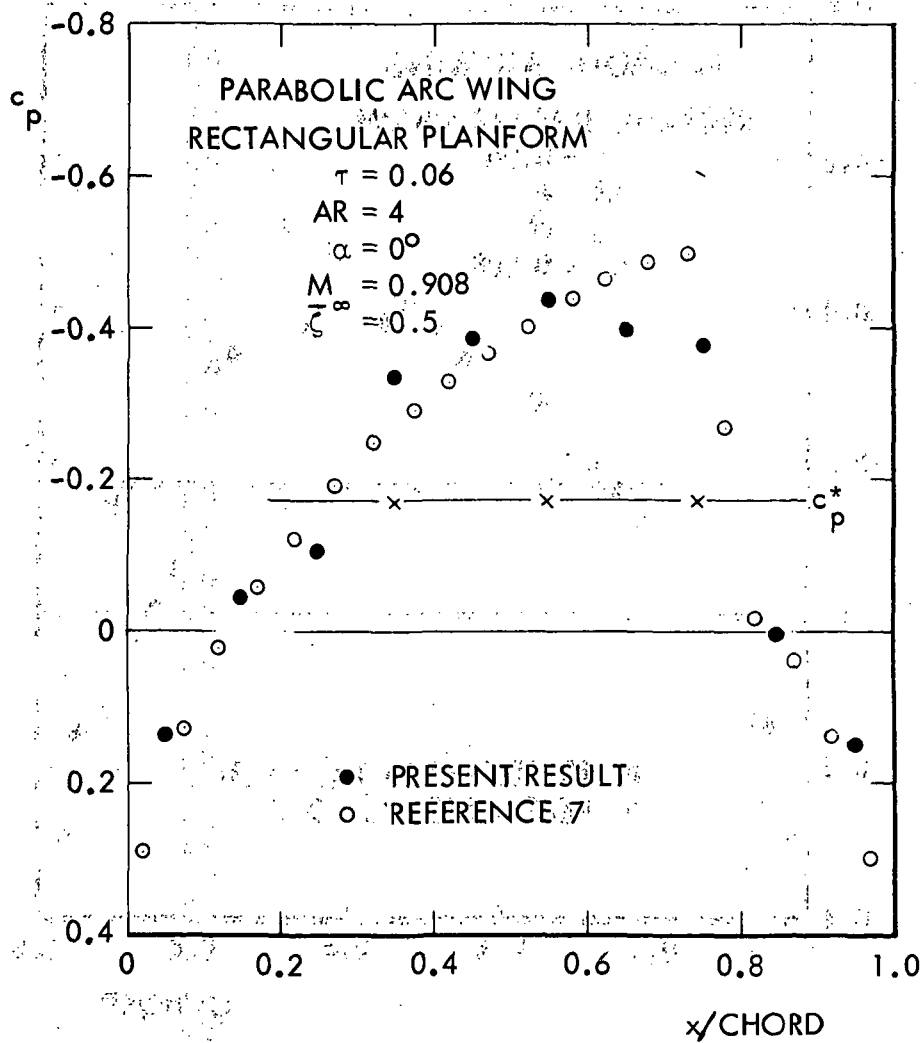


FIGURE 23 - Discontinuous flow past a non-lifting rectangular wing at $\bar{\zeta} = 0.5$.

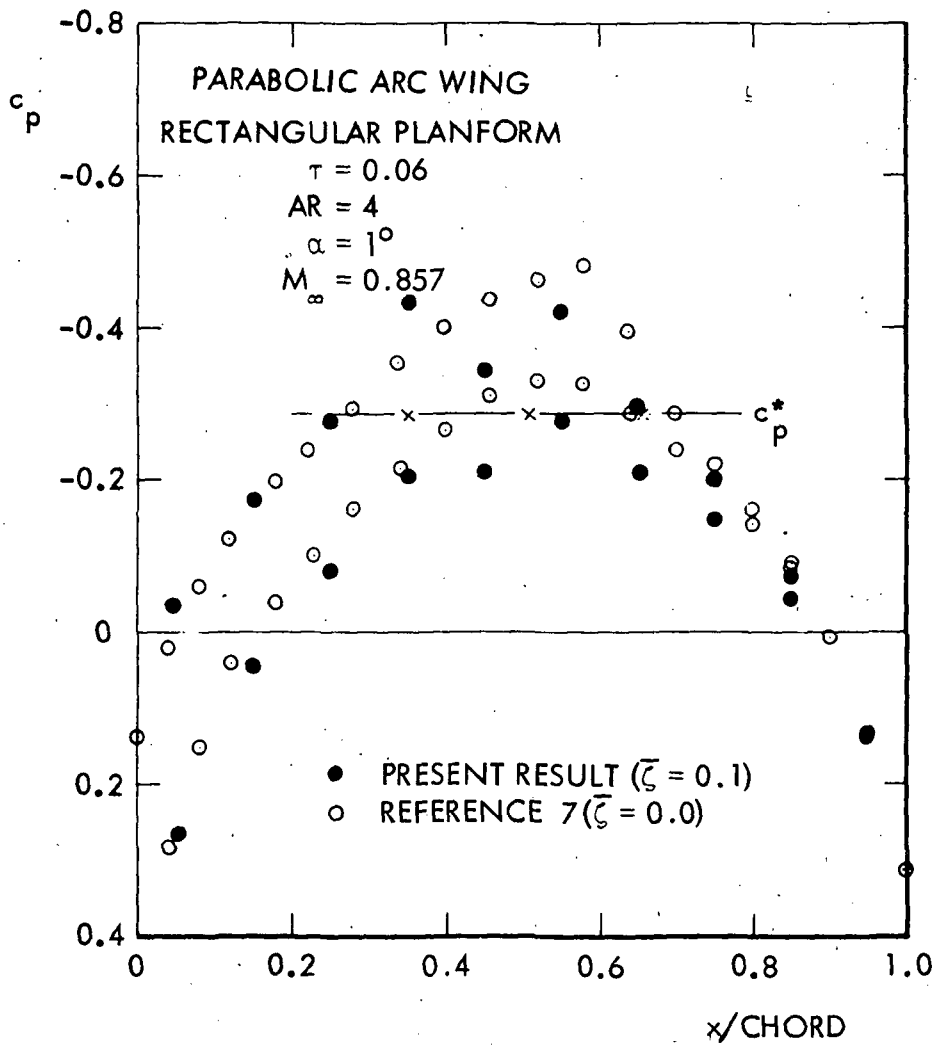


FIGURE 24 - Supercritical flow past a lifting rectangular wing near the mid-span station.

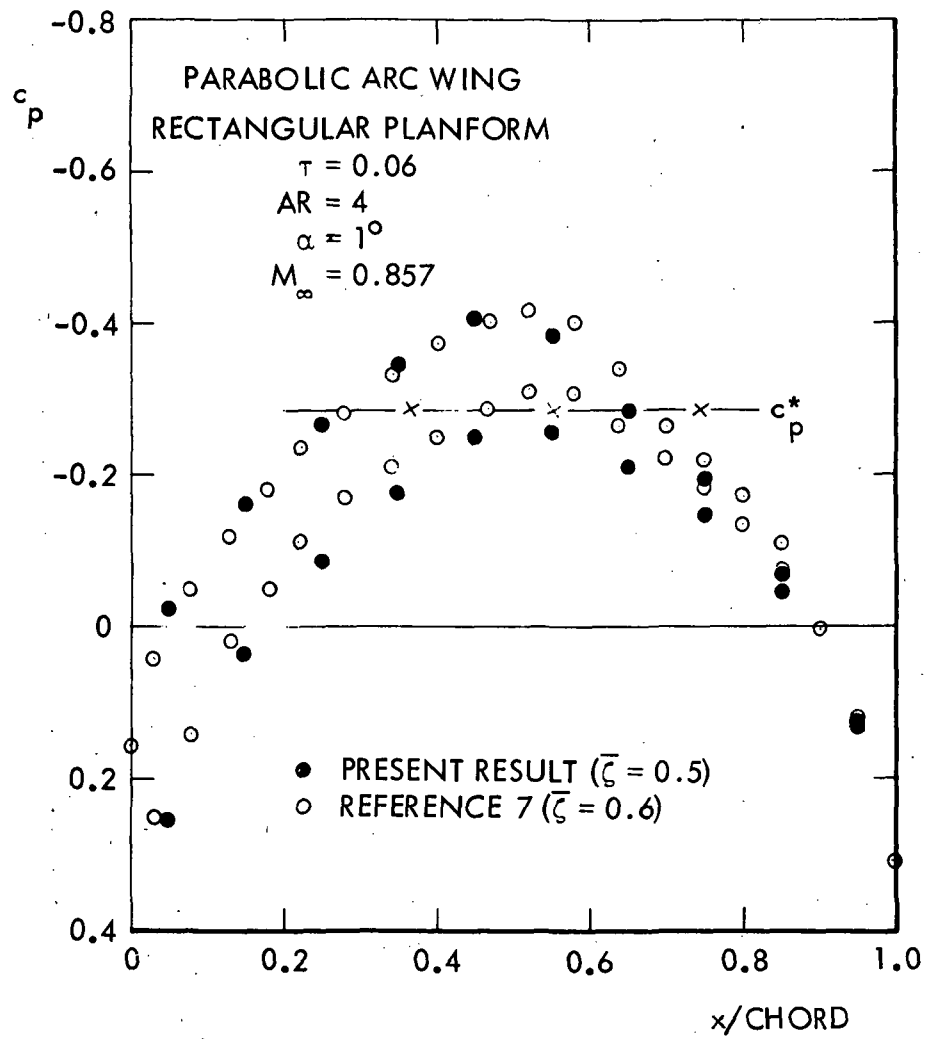


FIGURE 25 - Supercritical flow past a lifting rectangular wing.

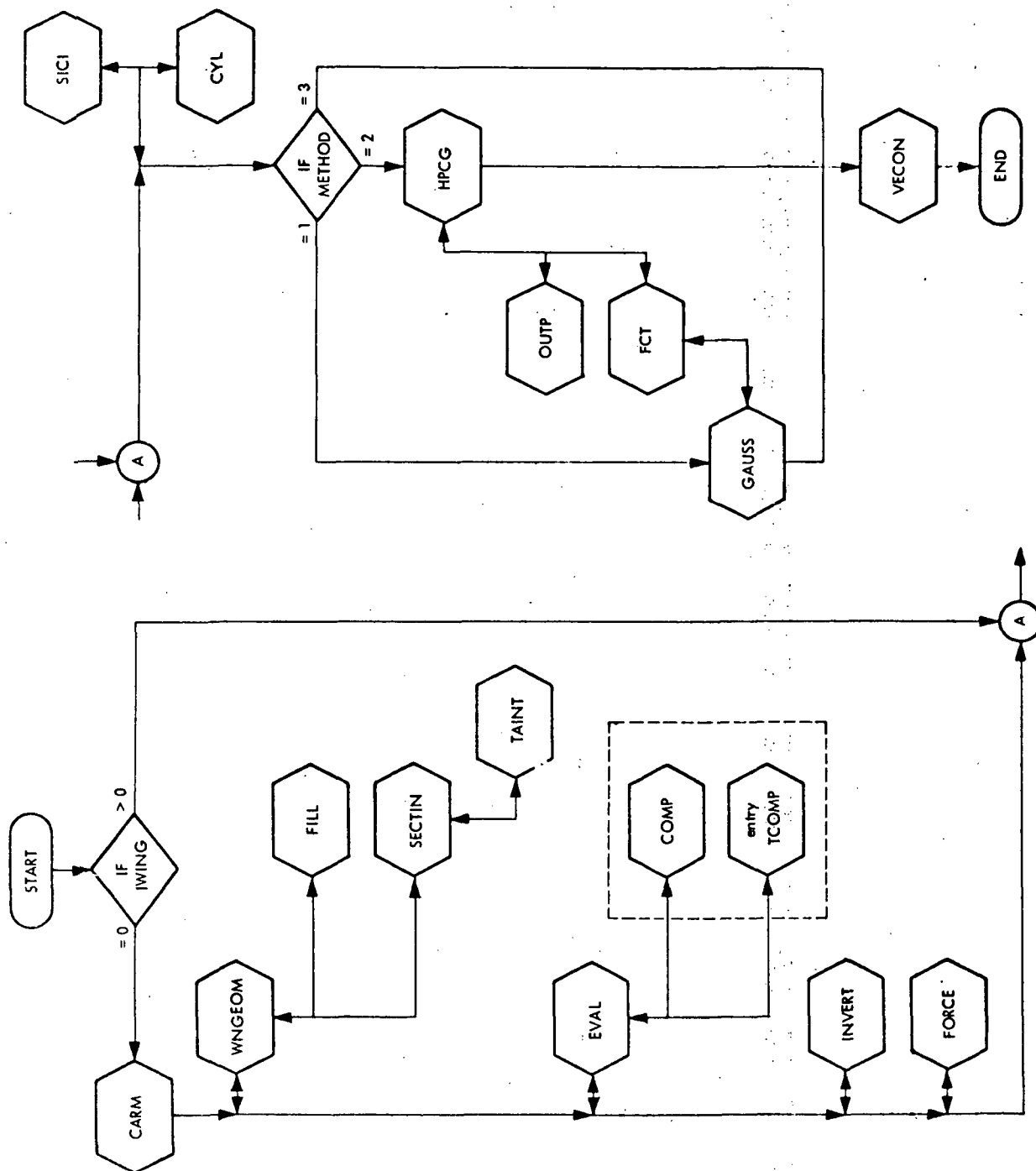


FIGURE 26 - Flow chart for the calling sequence of the program subroutines.

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